An example to interdisciplinary applications of fuzzy logic in plant communities

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Abstract

In this work, it is focused on the applicability of classifying and determining plant communities by using fuzzy logic in plant sociology. The advantages of fuzzy logic approach rather than classical methods are also given. Here it is introduced a logical and mathematical description method that can be used in phytosociological analyses. It is proposed that the vagueness of vegetation units can be clarified with *Fuzzy Set Theory (FST)*. Here, plant community type (ct) is expressed as a linguistic variable. A set of its linguistic values (S_{ct}) contains names of syntaxa (as primary terms) and some compound terms. Identifying a plant community is based on its floristic composition and assigning the most appropriate terms in S_{ct} to this plant community. It is presented the differences between taxa and syntaxa. And also the mathematical application of the variable ct in expert systems is discussed. Because *FST* is based on multi-valued logic, *FST* is better suited as a conceptual basis for vegetation science than two-valued logic (Aristotele's logic). This new method flips the disadvantages of classical Braun-Blanquet's method into advantages. It is much easier to separate plant communities by combining similar relevés with a fuzzy logic approach that provides more detailed information. Moreover, by applying fuzzy logic, it is possible to reveal the ecological structures of species, relevés, and plant communities. It becomes possible to analyse plant sociology data and reveal a relationship with ecological conditions applying computer programs developed by fuzzy logic. In a vegetation table prepared via Braun-Blanquet's method, it is likely encountered a whole set of data that contains very poor, vicious, relative, limited, and insufficient explanations. In contrast, in the current work, a vegetation table created by getting help from the fuzzy logic approach includes a detailed synthesis table containing the whole data set. In this approach, plant communities can be more rapidly separated and analysed. In the findings of vegetation studies, by "mathematical expressions"could provide needed clarity. With this study, the applicability of the fuzzy logic approach on plant sociology has been clearly demonstrated. Here it is proposed a novel plant sociology method that makes the results more understandable, objective, and useful.

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1 Introduction

The 21^{st} century is characterized by an increasing speed of technological developments. Nowadays, fuzzy logic and in general *Fuzzy Set Theory (FST)* (Zadeh, 1972) is used in many areas of technology

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and even in vegetation science (Dale, 1970; Andreucci et al., 2000; Wallace and Dale, 2005; Feoli et al., 2006; Zhang et al., 2006; Ochoa-Gaona et al., 2010; Feoli and Orlóci, 2011; Feoli and Zuccarello, 2013; Feoli, 2018; Hashemi, 2018; Carneiro et al., 2019; Mota et al., 2019). This new method flips the disadvantages of classical Braun-Blanquet's method into advantages. It is much easier to separate plant communities by combining similar relevés with a fuzzy logic approach that provides more detailed information. Moreover, by applying fuzzy logic, it is possible to reveal the ecological structures of species, relevés, and plant communities. With this study, the applicability of the fuzzy logic approach on plant sociology has been clearly demonstrated. Here it is proposed a novel plant sociology method that makes the results more understandable, objective, and detailed.

It is useful for the aim of this paper to recall that the term vegetation refers to a gathering of plants giving origin to plant communities, that can be defined at different hierarchical levels as vegetation types (Akman et al., 2001; Kılınç et al., 2005; Akdeniz Şafak, 2016). There are several methods for classifying vegetation. However the method of Braun-Blanquet (Braun-Blanquet, 1928; 1932; 1964) is the one that in these last years is widely applied cause of the fact that vegetation types are named and that they can be hierarchically classified in a syntaxonomical framework. This was of great utility to produce vegetation maps at different scales (Mueller Dombois and Ellenberg, 1974; Box and Fujiwara, 2005). Braun-Blanquet's method is based on floristic, ecological, and sociological data and has greatly contributed to the formation and spread of a branch of plant science called "plant sociology science". The proliferation and spread of vegetation studies have shown that plant sociology studies can also be done with different methods other than Braun Blanquet's method (Ewald, 2003).

Although Braun-Blanquet's method is widely used, it has been criticised because of its subjective structure (such as the selection of relevés (sampling areas) in field studies, in the application of the minimal area understanding, the studies in the relevés, the evaluation of the obtained data, the selection of characteristic and differential species, and its dependence on floristic data rather than ecological data). Braun-Blanquet's method aims to identify and unite the floristic, sociological similarities and group these to determine the plant communities. This classification method consists of a 5-stage tabulation process involving: the preparation of the pre (raw) table, the presence (constancy) table, the partial (fragment) table, the differentiated vegetation table, and the synthesis (synoptic = summary) tables (Table 1). This gives subjective results depending on the investigators and the ecological conditions. Also, the process is quite long, laborious, impractical and has a high degree of error likelihood (Mueller Dombois and Ellenberg, 1974; Box and Fujiwara, 2005). As noted, the classical Braun-Blanquet's method has long been criticized for its subjectivity. However, such numerical methods in recent years have eliminated these shortcomings.

In this joint work, we aimed to use fuzzy logic as mathematical modelling in order to eliminate the disadvantages of the Braun-Blanquet's method. Since fuzzy logic approach as a mathematical model to the phytosociological tables in a way that clarifies the ecological relationships among species, relevés, and plant communities by overwelming the vagueness of vegetation types (Moraczewski, 1993a; b). The vegetation types are stated as a linguistic variable (ct).

This new method flips the disadvantages of Braun-Blanquet's method into advantages. It is much easier to separate plant communities by combining similar relevés with a fuzzy logic approach that provides more detailed information. Moreover, by applying fuzzy logic, it is possible to reveal the ecological structures of species, relevés, and plant communities (Moraczewski, 1993a). It becomes possible to analyze plant sociology data and reveal a relationship with ecological conditions applying computer programs developed by fuzzy logic (Moraczewski, 1993b).

In a vegetation table prepared via Braun-Blanquet's method, it is likely encountered a whole set of data that contains very poor, vicious, relative, limited, and insufficient explanations. In contrast, in the current work, a vegetation table created by getting help from the fuzzy logic approach includes a detailed synthesis table containing the whole data set. In this approach, plant communities can be more rapidly separated and analyzed.

In the findings of vegetation studies, by "mathematical expressions" could provide needed clarity. Moraczewski has proposed that the fuzzy logic approach be applicable to plant sociology (Moraczewksi, 1993a, 1993b). Here it is proposed a novel plant sociology method that makes the results more understandable, objective, and useful.

FST is particularly effective for the identification of complex systems (Zadeh, 1972), including plant communities (Dale, 1970; Andreucci et al., 2000; Wallace and Dale, 2005; Feoli et al., 2006; Zhang et al., 2006; Ochoa-Gaona et al., 2010; Feoli and Orlóci, 2011; Feoli and Zuccarello, 2013; Feoli, 2018). Here in this joint work submits logical and numerical description methods for phytosociological analyses in which the vagueness of vegetation units has been clarified formal tools developed by *FST*.

The vegetation analysis has been offered as a linguistic variable plant community type (ct). A set of its linguistic values (S_{ct}) contains names of syntaxa (as primary terms) and some compound terms (Moraczewski, 1993a). According to the linguistic approach proposed here, plant communities are identified based on *floristic composition (FC)* - floristic composition of an area or a plant community composes from plant taxa - and involves assigning the most appropriate term in S_{ct} to this plant community. According to our literature review, this study seems to be the first interdisciplinary study in which graded logic has been applied in the field of vegetation based on field studies rather than theory.

2 Data and method

The database used in the study was taken from Akdeniz Şafak, 2016. Here, it is aimed to eliminate the ambiguity in syntaxonomic studies to improve the understandability and provide more realistic expressions by converting linguistic statements into numerical values via fuzzy logic (Damasceno et al., 2019).

The method used can be considered a linguistic approximation via fuzzy logic. The logical differences between taxa, syntaxa, and the numerical application of the variable (ct) have been already discussed elsewhere (Moraczewski, 1993a).

The three syntaxa are indicated as AS_1 or AS_2, and AS_3 (see Table 1 below). In the used formulation AS_i is defining the ith plant community, and r_j is a relevé in a given plant community. Set AS is a subset of S_{ct}. $FC(p)$ is the floristic composition of p; where p (phytocoenose), phytocoenose means plant community, shows any relevés and plant group (plant community, association, alliance, order, classis, etc.). The identification of a plant community and definition of its name is expressed as $FC(p)$. $FC(r_j)$ is the floristic composition of r_j. ChSC_i is the characteristic species composition of the ith plant community.

Plant community type as a linguistic variable and translating into mathematical modelling

In the classification (typology) of plant sociology, the ambiguities of syntaxa and the names of plant communities and opinions about them can only be truly formally expressed by assigning linguistic variables to them. (For details see Moraczewski, 1993a). For the sake of building Table 2, here it

is need the following basic concepts: A linguistic variable is a quintuple (ct, S_{ct}, R, G, M) where ct is the name of the variable; S_{ct} is the term set of ct - the linguistic values; R is the universe of discourse; G is the syntactic rule that consists of verbal expressions and generates the terms in S_{ct}; and M is a semantic rule that assigns a meaning M(AS_i) – a fuzzy set in R for each linguistic value of AS_i. For example, $S_{ct} = \{AS_i$, an impoverished fragment of AS_i, closest to AS_i, not AS_i, transitional between AS_i and AS_j, indeterminate$\}$ where syntaxa represents the variable ct – a linguistic variable with values defined in the term set S_{ct}. The universe of discourse is R = [AS_i, indeterminate]. The syntactic rule defines the rule for linguistic values to be generated, and the semantic rule defines the membership function for each linguistic value. Linguistic variables can be extended to a more complex form through operators such as negation, assertion, reinforcement, hedge, logical, and connective (see also Zadeh, 1975; Yen, 1999).

Table 1. Phytosociological table of 21 relevés $(r_1, ..., r_{21})$, in which 10 taxa $(a, ..., j)$ were found. Here in this table mathematical modelling in crisp sense.

	Code and Name of syntaxon	AS_1 (*Carduo-Diplotaxietum tenuifoliae*)							AS_2 (*Anchuso-Resedetum luteae*)							AS_3 (*Galio-Medicaginetum minimae*)						
	Code of relevé	r_1	r_2	r_3	r_4	r_5	r_6	r_7	r_8	r_9	r_{10}	r_{11}	r_{12}	r_{13}	r_{14}	r_{15}	r_{16}	r_{17}	r_{18}	r_{19}	r_{20}	r_{21}
Code of Taxon	Taxa / Relevé number	61	62	63	64*	65	66	68	17	18	21	22	28	32	51*	95	96	97	98	100	101	102*
a	*Carduus pycnocephalus ssp. albidus*	1	1	1	1	.	1	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.
b	*Diplotaxis tenuifolia*	.	1	1	1	1	1	1	.	.	.	.	.	.	.	.	.	.	.	.	.	.
c	*Valerianella coronata*	1	.	.	.	1	.	1	.	.	.	.	.	.	.	.	.	.	.	.	.	.
d	*Reseda lutea var. lutea*	.	.	.	.	.	.	.	1	1	1	1	1	1	1	.	.	.	.	.	.	.
e	*Anchusa azurea*	.	.	1	1	1	1	.	.	.	1	1	.	.	.	.	.	.	.	.	.	.
f	*Galium verum*	.	1	1	1	.	1	.	.	.	.	.	1	1	.	1	1	1	1	1	1	1
g	*Medicago minima var. minima*	1	.	.	.	.	.	1	.	.	.	.	.	1	.	1	1	.	1	1	1	1
h	*Erodium cicutarium*	.	1	1	1	1	.	1	.	.	.	.	.	.	.	.	.	.	.	.	.	1
i	*Bromus tectorum*	.	.	.	.	.	.	1	1	1	1	1	.	.	.	.	.	1	.	.	.	.
j	*Capsella bursa-pastoris*	1	.	.	.	1	1	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.

*Holotip relevés: These relevé represent the communities, given the above Table 1, best.

In this work, R is the set of all relevés in the research area. This contains 21 elements (a set of 21 relevés), and the elements of R are symbolized by r_1, r_2, ..., r_{21}. The linguistic values of syntaxon AS_i are shown in column G, and the rules of the calculations for the degree of truth to fuzzy values are shown in column M. The degrees gradually go down from the biggest values to the smallest values (Table 2). Syntaxon AS_i and its fuzzy values are given in Table 3. In Table 2, the necessary formulas to fuzzy the crisp values in Table 1 are given. As for Table 3, the crisp values in Table 1 are fuzzy with the help of Table 2.

The meaning of any term AS_i in AS is the syntaxon AS_i - a fuzzy set in R defined as follows:

$$AS_i = ((Z_i)^a)^N, \mu_{zi}(r_j) = I(ChSC_i, FC(r_j)), \qquad i = 1, ..., 3; \qquad j = 1, \ldots, 21,$$

where $((Z_i)^a)^N$ is normalized ath power of $Z_i \subset R$ $(a \geq 1)$, and $I(ChSC_i, FC(rj))$ is the grade of

inclusion of the crisp set $ChSC_i$ in $FC(r_j)$. Z_i is the set of all relevés in a syntaxon (e.g. plant community).

Here, we assume that the grade of inclusion (Jaccard's similarity coefficient) is defined as

$$I(X,Y) = |X \cap Y|/|X|$$

where $|.|$ denotes cardinality and X, Y are crisp sets (See also Moraczewski, 1993a).

The fuzzy logic approach is used – which overlaps with the methods of Braun-Blanquet (Braun-Blanquet, 1932) used in classical plant sociology research – generalizes the results further.

The relevés which are taken during the field studies in this research area and the taxa included in these relevés are given in Table 1, together with the classical (crisp) situation;

In Table 1, "1" is the numerical value of "*present*" and also "the points" (.) refers to "*absent*". In the linguistic sense, "1" means "*there is*", and "." indicates that it is "*there is not*". So, the compatible equivalence of linguistic values are "0" and "1" in crisp sense.

Now, here it can given the two parts (G and M) of the quintuple (ct, S_{ct}, R, G, M) in Table 2.

Table 2. Syntactic rule and corresponding semantic rule of the linguistic variable community type. AS_i and AS_j are the names of syntaxa AS_i and AS_j; $i, j = 1, \ldots, 3$.
Here in this table mathematical modelling in fuzzy sense.

Syntactic rule (G)	**Semantic rule (M)**
$AS_i \in S_{ct}$	M $(AS_i) = AS_i = ((Z_i)^{a})^{N}$
"an impoverished fragment of AS_i" $\in S_{ct}$	M ("an impoverished fragment of AS_i") = $(AS_i)^{b}$, $b > 1$
"closest to AS_i" $\in S_{ct}$	M ("closest to AS_i") = $(AS_i)^{c}$, $c < 1$
"not AS_i" $\in S_{ct}$	M ("not AS_i") = $\neg\, AS_i$
"transitional between AS_i and AS_j " $\in S_{ct}$ i ≠ j	M ("transitional between AS_i and AS_j") = $(AS_i \cup AS_j)^{d}$, i ≠ j, $d > 1$
"indeterminate" $\in S_{ct}$	M ("indeterminate") = $\{O/r_k\}$, $k = 1,\ldots,21$

Here, the power values can be taken as $a = 1.25$, $b = 4$, $c = 0.5, d = 2$ (see also Moraczewski 1993a).

Identifications of plant communities

$\mu_z(r_j) = JS(FC(p), FC(r_j))$, $j = 1, ..., 21$ denotes the Jaccard's similarity coefficient between $FC(p)$ and $FC(r_j)$. Here, Jaccard's similarity coeffient was used in order to identification of names of plant communities. In this work, preferred Jaccard's similarity index because it has been used in similar studies with its simplicity and practicality (for details see Goodall 1973 and Moraczewski 1993a).

This function adjoins a name to every phytocoenose denoted by DET.

$DET(p) = LA_{ct}(Z^N)$, $\forall p \in P$ LA: Linguistic Approximation, Z^N: The normalized set Z calculated for the phytocoenose p. Determining the name of a plant community p at $FC(p) = \{a, b, c\}$. The transition from $FC(p)$ to $DET(p)$ is illustrated in Figure 2.

3 Results and discussion

The article of Moraczewski (1993a), main cited reference of this work, is based on artificial plant communities. Thankfully, in our joint work it is addressed real plant communities determined by field studies.

A syntaxon is generally defined as a plant community (unit of abstract plant sociology with no fixed rank) (Westhoff and van der Maarel, 1973; Mirkin et al., 1989; Moraczewski, 1993a). It should be noted that

the *FST* provides a very convenient and logical method of describing the complexity of phytosociological analysis and hierarchy.

The main logical differences between taxon and syntaxon are, first of all, taxa are generally regarded as ordinary (crisp) sets (cf. Gregg, 1954; Buck and Hull, 1966), but syntaxa should be considered as fuzzy sets. The names of syntaxa are unclear; therefore, fuzzy naming arrangements can be used, but taxon names are precise, and no fuzzy naming arrangements are necessary. For instance, as a taxon name "*Carduus nutans*" (in Turkish: Eşekdikeni) is the classic (crisp) set element. But the name of a plant community is open to interpretation. As a syntaxon name "*Carduo-Diplotaxietum tenuifoliae*" may be given. There is a deeper reason for this linguistic difference: the intangible vegetation area is more complex than the classical character area (Whittaker, 1962; Matuszkiewicz, 1981; Mirkin, 1989). The degree of fuzziness of syntaxa depends on the rate of continuity and discontinuity observed in nature. The greater the discontinuity, the better the vegetation is divided into its constituent units (Whittaker, 1962; Goodall, 1973; Westhoff And van der Maarel, 1973; van der Maarel, 1975; Moravec, 1989).

The situation in vegetation science is more complicated. According to the approach presented above ($cf.$ Zadeh, 1983), the number of elements of a syntaxon S described is not more than the number of plant communities that form a basis for syntaxonomic synthesis. Although each plant population can be interpreted syntaxonomically correct and complete (Matuszkiewicz, 1981), it may not be evaluated under any syntaxa. For this reason, vegetation analysis and hierarchy should be regarded as a very complex intangible system with syntaxa and their own naming and meaning rules where their names are used. Establishing a connection between syntaxa names and syntaxa themselves, considered finite fuzzy sets, allows 'calculation' of the optimal name for any plant community. This assigns any element of P a name from the term sentence of the variable plant community type.

Fuzzy sets, which are the meanings of simple and compound names of plant communities, are defined in this study depending on their Characteristic Species Combinations (ChSC). In this study, rather than determining the syntaxonomic level of a syntaxon, it is attempted to identify and name that syntaxon itself. However, to better explain the logical structure of the syntaxonomic hierarchy related to this subject, future studies should address specific sub-topics in greater detail. *FST* provides a convenient and logical method for eliminating the complexity of phytosociological analysis and hierarchy. For example, "the fact that vegetation units are in a multi-faceted and complex relationship with each other rather than hierarchical systems" (van der Maarel, 1989) can be better expressed with the understanding of classification according to fuzzy set theory.

Only the *FST* can provide common ground between the degree of specialization of a plant sociologist and a computer ($cf.$ Zadeh, 1978; Zadeh, 1983). Suppose a computer must deal with the syntaxa information of a field study. In that case, the computer must know not only the further full diversity of the primary and regulated names of the vegetation units but also the meanings. In other words, a computer should be able to comprehend almost all the details or differences between the expressions "*Carduo-Diplotaxietum tenuifoliae*" and "closest to-*Carduo-Diplotaxietum tenuifoliae*". A computer that works with fuzzy logic is capable of detecting such uncertain matters. The examples show that the proposed syntactics (G) and semantics (M) correspond to estimates of authors of this work. Technical details on building knowledge bases by means of linguistic variables can be found in Zadeh (1978, 1983).

This study is based on many theoretical aspects in vegetation science, particularly on different perspectives concerning phytosociological generalizations, predictions, and facts. That is working with fuzzy logic in plant sociology and ecology has advantages. These works will lead for the future studies on fuzzy logic in plant sociology and ecology And about Identifications of Plant Communities,

The fuzzified results obtained with the calculations made according to the data in Table 1 are presented in Table 3. And also, it is presented fuzzy values of S_{ct} as a graph, such as it has been done in Figure 1.

Table 3. Fuzzy sets that are meaning of terms in S_{ct} (according to Table 1) (for all syntaxa)
Abbreviations : *imp.* AS_i= an impoverished fragment AS_i, $+/-AS_i$= closest to AS_i, AS_i & ASj = transitional between AS_i and AS_j, indet. = indeterminate.

Code and name of syntaxon	AS_1 (Carduo-Diplotaxietum tenuifoliae)							AS_2 (Anchuso-Rezedetum luteae)							AS_3 (Galio-Medicaginetum minimae)						
Code of relevé	r_1	r_2	r_3	r_4	r_5	r_6	r_7	r_8	r_9	r_{10}	r_{11}	r_{12}	r_{13}	r_{14}	r_{15}	r_{16}	r_{17}	r_{18}	r_{19}	r_{20}	r_{21}
Taxa relevé number	61	62	63	64*	65	66	68	17	18	21	22	28	32	51*	95	96	97	98	100	101	102*
AS_1	0.6	0.6	0.6	1	0.6	0.6	0.6	0	0	0	0	0	0	0	0	0	0	0	0	0	0
AS_2	0	1	0.42	0.42	0.42	0.42	1	0.42	0.42	1	1	0.42	0.42	0.42	0	0	0	0	0	0	0
AS_3	0.42	0.42	0.42	0.42	0.42	0	0.42	0	0	0	0	0.42	1	0	1	1	0.42	1	1	1	1
$imp.\ AS_1$	0.13	0.13	0.13	1	0.13	0.13	0.13	0	0	0	0	0	0	0	0	0	0	0	0	0	0
$imp.\ AS_2$	0	1	0.03	0.03	0.03	0.03	1	0.03	0.03	1	1	0.03	0.03	0.03	0	0	0	0	0	0	0
$imp.\ AS_3$	0.03	0.03	0.03	0.03	0.03	0	0.03	0	0	0	0	0.03	1	0	1	1	0.03	1	1	1	1
$\neg.AS_1$	0.77	0.77	0.77	1	0.77	0.77	0.77	0	0	0	0	0	0	0	0	0	0	0	0	0	0
$\neg.AS_2$	0	1	0.65	0.65	0.65	0.65	1	0.65	0.65	1	1	0.65	0.65	0.65	0	0	0	0	0	0	0
$\neg.AS_3$	0.65	0.65	0.65	0.65	0.65	0	0.65	0	0	0	0	0.65	1	0	1	1	0.65	1	1	1	1
$Not.AS_1$	0.4	0.4	0.4	0	0.4	0.4	0.4	1	1	1	1	1	1	1	1	1	1	1	1	1	1
$Not.AS_2$	1	0	0.58	0.58	0.58	0.58	0	0.58	0.58	0	0	0.58	0.58	0.58	1	1	1	1	1	1	1
$Not.AS_3$	0.58	0.58	0.58	0.58	0.58	1	0.58	1	1	1	1	0.58	0	1	0	0	0.58	0	0	0	0
$AS_1\&AS_2$	0.36	0.36	0.36	1	0.36	0.36	1	0.18	0.18	1	1	0.18	0.18	0.18	0	0	0	0	0	0	0
$AS_1\&AS_3$	0.36	0.36	0.36	1	0.36	0.36	0.36	0	0	0	0	0.18	1	0	1	1	0.18	1	1	1	1
$AS_2\&AS_3$	0.18	1	0.18	0.18	0.36	0.18	1	0.18	0.18	1	1	0.18	1	0.18	1	1	0.18	1	1	1	1
Indet.	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0

Table 4. Names assigned to plant community of various floristic compositions.

$FC(p)$	$DET(p)$	$FC(p)$	$DET(p)$
$\{a,b,c\}$	AS_1	$\{a,b,c\}$	$imp.AS_1$
$\{d,e\}$	AS_2	$\{d,i\}$	$imp.AS_2$
$\{f,g\}$	AS_3	$\{g,j\}$	$imp.AS_3$
$\{a,b,h\}$	AS_1	$\{h\}$	indet.
$\{d,i\}$	AS_2	$\{i\}$	indet.
$\{f,g,h\}$	AS_3	$\{j\}$	indet.
$\{a,b,j\}$	AS_1	$\{h,i\}$	indet.
$\{d,e,g\}$	AS_2	$\{h,j\}$	indet.
$\{f,g,i\}$	AS_3	$\{i,j\}$	indet.
$\{a,b,c,e,f,g,h,i,j\}$	AS_1	$\{h,i,j\}$	indet.
$\{d,e,f\}$	AS_2	$\{f,g,i\}$	$+/-AS_1$
$\{f,g,h,i\}$	AS_3	$\{f,g,i\}$	$+/-AS_2$
$\{a,b,e\}$	AS_1	$\{e,f,g,i\}$	$+/-AS_3$
$\{d,e,i\}$	AS_2	$\{e,f,g,i\}$	$AS_1\&AS_2$
$\{a,b,f\}$	AS_1	$\{f,g,h,i\}$	$AS_1\&AS_3$
$\{d,e,g,i\}$	AS_2	$\{f,g,i\}$	$AS_2\&AS_3$
$\{d,e,f,i\}$	AS_3		

FC(p): Floristic composition of phytocoenose p; *DET (p)*: Determining the name of community ASi; i.e. *DET (AS1)*: Determining the name of a community *AS1* at *FC(AS1)*=$\{a,b,c\}$. The function that adjoins a name to every phytocoenose denoted by *DET*. *Abbreviations:* imp. AS_i= an impoverished fragment AS_i, $+/-AS_i$= closest to AS_i, AS_i & ASj = transitional between AS_i and AS_j, indet. = indeterminate.

1st step: Jaccard's similarity coefficient between phytocoenose p and each relevé from R is calculated. So the obtained fuzzy set Z is normalized.

2nd step: The linguistic approximation (LA) (finding the term whose meaning is least distant from Z^N) (Moraczewski, 1993a). In this case, in this study, the name of the new syntaxon is "*Carduo-Diplotaxietum tenuifoliae*". In short, notation is AS_1. Thus, $DET(p) = AS_1$ is obtained. (Figure 2 and Table 4).

Advantages of the proposed method (a new mathematical modelling) for plant communities.

This study aimed to see more of the data in Table 3 than in Table 1. Indeed, despite the information given in Table 1 as "present (1)" or "absence (0)", explaining this situation with mathematical values in Table 3 made the study more meaningful. The fuzzy approach can reveal the natural model as much as possible because it gives the necessary details in explaining the work results. With the values of membership

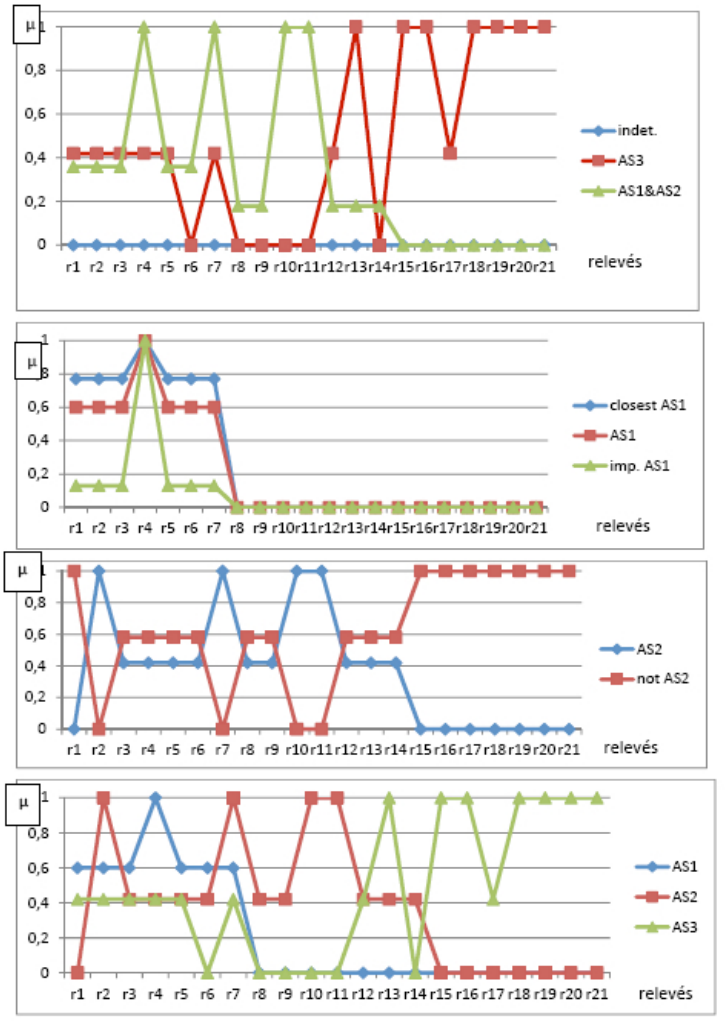


FIGURE 1. Membership functions of fuzzy sets which are meanings of some terms in S_{ct}. Abbreviations as in Table 3.

degrees given in Table 3, information about which taxon is effective, dominant, and permanent in which plant community and relevés (sampling areas) is much better understood than Table 1. Likewise, it is possible to generate answers to questions such as "how close are the plant associations (plant community) to each other and how similar are they?".

Similarly, such questions can be answered in Figure 1, which is a different representation of Table 3. In the classical explanation of the syntaxonomy, the result is expressed as in Table 1. Considering the situation in Table 1, the results match the classical sentence understanding, but in this approach, the subject begins to take shape exactly, and Table 3, which is formed by the calculation of values, is the most expressive

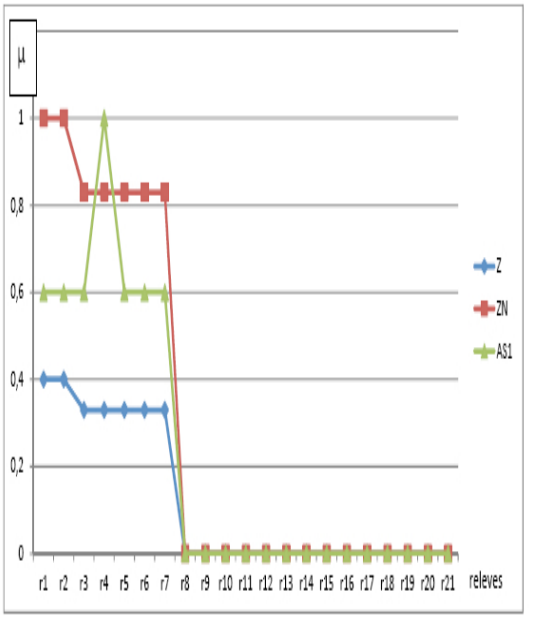


FIGURE 2. DET (AS_1): Determining the name of a community AS_1 at $FC(AS_1) = \{a, b, c\}$. Where Z is the set of all relevés in a syntaxon and Z^N is normalized set of Z.

presentation of the syntaxonomic units. The advantages of Table 3 relative to Table 1 can be understood, as explained in the introduction.

For example, it can be seen in Table 1 that AS_1 does not have anything in common with AS_2 and AS_3. However, in Table 3, the degree of this dissimilarity and the comparison between AS_1, AS_2, and AS_3 are clearly seen together with the membership degree AS_2 and AS_3 are more similar than AS_1. Similar comparisons can be understood by looking at Tables 1 and 3 for linguistic variables.

The results in Table 3 are obtained from the above calculations (for those interested in which equations were made with calculations, see Moraczewski, 1993a).

Expected achievements

It is hoped that this work will be useful for plant sociology scientists. It should provide relief for these researchers in this field and provide a more realistic expression of uncertainty, ambiguity, and unknown statements. Furthermore, it will provide a better understanding of plant sociology studies by providing meaningful and realistic expressions.

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References

[1] Ş.S. Akdeniz, *Syntaxonomical analysis of the ruderal vegetation in province Eskişehir.* The PhD Thesis of the Institute of Natural and Applied Sciences, Department of Biology, Ankara University. 102 pp (In Turkish) (2016), TR.

[2] Ö. Akın, M.Ü. Bingöl and Ş.S. Akdeniz, *Application of Fuzzy Logic And Fuzzy Similarity in Plant Sociology. Internatıonal Symposium on Applied Mathematics And Engineering* (ISAME-2022), 21-23 January, Biruni University, Istanbul, Turkey. https://ntmsci.com/Conferences/ISAME2022,-https://ntmsci.com/ISAME_2022_Final_Program.pdfi, https://www.facebook.com/517286006/videos/604915997467407/ (Presentation time: 5: 54), (Online Invited Keynote Oral Presentation).

[3] Y. Akman, O. Ketenoğlu and F. Geven, *Vegetation Ecology and Research Methods.* Palme Publication, Ankara (2001) (In Turkish). 341 pp.

[4] F. Andreucci, E. Biondi, E. Feoli and V. Zuccar, *Modeling environmental responses of plant associations by fuzzy set theory.* Community Ecology 1 (2000), 73-80. https://doi.org/10.1556/ComEc.1.2000.1.10.

[5] E.O. Box and K. Fujiwara, *Vegetation types and their broad-scale distribution.* In: van der Maarel E (Ed.) Vegetation ecology. Blackwell, Oxford (2005), pp. 106–128.

[6] J. Braun–Blanquet, *Pflanzensoziologie.* Grundzüge der Vegetationskunde. Biologische Studienbücher 7.1. Ed., (1928) Berlin. X + 330 pp.

[7] J. Braun–Blanquet, *Plant sociology.* (transl. by G. D. Fuller and H. S. Conard), Mc Graw Hill, New York and London (1932), XVIII + 439 pp.

[8] J. Braun–Blanquet, *Pflanzensoziologie. Grundzüge der Vegetationskunde.* 3rd ed. Springer, Wien – New York (1964). 865 pp.

[9] B.G. Buchanan, D. Barstow, R. Bechtel, J. Bennett, W. Clancey, C. Kulikowski, T. Mitchell and D.A. Waterman, *Constructing an expert system.* In: F Hayes-Roth, DA Waterman & DB Lenat (Eds.), Building Expert Systems. Reading, MA: Addison-Wesley Publishing Company. London (1983). pp. 127-167.

[10] R.C. Buck and D.L. Hull, The logical structure of the linnaean hierarchy. Systematic Zoology 15 (1966), 97-111.

[11] A.R.T. Carneiro, D.A. Sanglard, A.M. Azevedo, T.L.P.O.D. Souza, H.S. Pereira and L.C. Melo, *Fuzzy logic in automation for interpretation of adaptability and stability in plant breeding studies.* Scientia Agricola 76 (2019), 123-129.

[12] B.R.S. Costa, H. Oldoni, R.C. Rocha and L.H. Bassoi, *Delimitation of homogeneous zones in vineyards using geostatistics and multivariate analysis of different vegetation indices.* Engenharia Agrí cola, Jaboticabal, special issue 39 (2019), 13-22. https://doi.org/10.1590/1809-4430-Eng.Agric.v39nep13-22/2019

[13] F.A. Damasceno, C.E. Alves Oliveira, L.H. Pedrozo Abreu, J.A. Osorio Saraz and P.F.P. Ferraz, *Fuzzy system to evaluate performance and the physiological responses of piglets raised in the farrowing house with different solar heating systems.* Revista Facultad Nacional de Agronomía Medellín 72 (2019), 8729-8742.

[14] M.B. Dale, *Systems analysis and ecology.* Ecology 51 (1970), 2-16.

[15] J. Ewald, *A critique for phytosociology.* Journal of Vegetation Science, 14 (2003), 291-296.

[16] E. Feoli, G. Ferro and P. Ganis, *Validation of phytosociological classifications based on a fuzzy set approach.* Community Ecology 7 (2006), 99-108. https://doi.org/10.1556/ComEc.7.2006.1.10.

[17] E. Feoli and L. Orlóci, *Can similarity theory contribute to the development of a general theory of the plant community?.* Community Ecology 12 (2011), 135–141.

[18] E. Feoli and V. Zuccarello, *Fuzzy sets and eigenanalysis in community studies: classification and ordination are two faces of the same coin.* Community Ecology 14 (2013), 164-171. https://doi.org/10.1556/ComEc.14.2013.2.6.

[19] E. Feoli, *Classification of plant communities and fuzzy diversity of vegetation systems.* Community Ecology 19 (2018), 186-198. https://doi.org/10.1556/168.2018.19.2.11.

[20] D.W. Goodall, *Sample similarity and species correlation.* In: Whittaker, RH. (ed.), Handbook of Vegetation Science. Part V. Ordination and classification of communities, W. Junk, The Hague (1973). pp 105-156.

[21] J.R. Gregg, *The language of taxonomy.* Columbia University Press (1954), New York.

[22] S.A. Hashemi, *Ecological capability evaluation for afforestation and forest expansion using Geographic Information System (GIS) in management area of Caspian Sea.* Anais da Academia Brasileira de Ciências 90 (2018), 3761-3768.

[23] M. Kılınç, *Plant sociology (Vegetation Science).* Palme Publications & Bookshops Ltd. Co. 284 p, Ankara (2005). (In Turkish).

[24] W. Matuszkiewicz, *A guide for identification of the plant communities of Poland.* PWN, Warszawa (1981) (in Polish).

[25] M.T. Mota, A. Bressane, J.A.F. Roveda and S.R.M.M. Roveda, *Classification of successional stages in atlantic forests: a methodological approach based on a fuzzy expert system.* Ciencia Florestal, Santa Maria, 29 (2019), 519-530.

[26] B.M. Mirkin, *Plant taxonomy and syntaxonomy: A comparative analysis.* Vegetatio 82 (1989), 35-40.

[27] B.M. Mirkin, G.S. Rozenberg and L.G.Naumova, *The dictionary of modern phytocoenology.* Nauka, Moskva (1989) (in Russian).

[28] I.R. Moraczewski, Fuzzy logic for phytosociology 1. Syntaxa as vague concepts, Vegetatio 106: 1-11.

[29] Moraczewski, I.R. (1993b). *Fuzzy logic for phytosociology 2.* Generalizations and prediction, Vegetatio 106 (1993), 13-20.

[30] J. Moravec, *Influences of the individualistic concept of vegetation on syntaxonomy.* Vegetatio 81 (1989), 29-39.

[31] D. Mueller-Dombois and H. Ellenberg, *Aims and Methods of Vegetation Ecology.* John Wiley and Sons, New York, 547 p. Noble IR 1987. The role of expert systems in vegetation science. Vegetatio 69 (1974), 115-121.

[32] S. Ochoa-Gaona, D.E. Kampichler, B.H.J. Jonga, S. Hernández, V. Geissena and E. Huertaa, *A multi-criterion index for the evaluation of local tropical forest conditions in mexico.* Forest Ecology and Management 260 (2010), 618-627.

[33] I. Şafak, Fuzzy logic and its application areas in forestry. Journal of South-West Anatolia Forest Research Institute 9: 1-30. (In Turkish).

[34] E. van der Maarel, *The Braun-Blanquet approach in perspective.* Vegetatio 30 (1975), 213-219.

[35] V. Westhoff and E. van der Maarel, *The Braun-Blanquet approach.* In: Whittaker. RH (ed.), Handbook of Vegetation Science. Part V. Ordination and Classification of Communities (1973), pp 617-726. W. Junk, The Hague.

[36] C.S. Wallace and MB Dale, *Hierarchical clusters of vegetation types.* Community Ecology 6 (2005), 57-74. https://doi.org/10.1556/ComEc.6.2005.1.7.

[37] R.H. Whittaker, *Classification of natural communities.* The Botanical Review 28 (1962), 1-239.

[38] V.C. Yen, *Rule selections in fuzzy expert systems.* Expert Systems with Applications 16 (1999), 79-84.

[39] L.A. Zadeh, *Quantitative fuzzy semantics.* Information Sciences 3(2) (1971), 159-176.

[40] L.A. Zadeh, *A new approach to system analysis.* In: Marois, M. (Ed.), Man and Computer. North-Holland, Amsterdam (1972).

[41] L.A. Zadeh, *The concept of a linguistic variable and its application to approximate reasoning.* Part I, Information Sciences 8, 199-249; Part II, Information Sciences 8, 301-357; Part III, Information Sciences 9 (1975), 43-80.

[42] L.A. Zadeh, PRUF - *A meaning representation language for natural languages.* Int. J. Man-Machine Studies 10 (1978), 395-460.

[43] L.A. Zadeh, *The role of fuzzy logic in the management of uncertainty in expert systems.* Fuzzy Sets and Systems 11 (1983), 199-227.

[44] J. Zhang, Y. Xi and J. Li, *The relationships between environment and plant communities in the middle part of Taihang Mountain Range, North China.* Community Ecology 7 (2006), 155-163. https://doi.org/10.1556/ComEc.7.2006.2.