Weighted sharing and uniqueness for the shifts of meromorphic functions*

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Abstract

In this paper, we deal with the uniqueness problem for q-shifts of meromorphic functions. The results obtained in this paper are the q-shift analogues of theorems given by Yang and Zhang [Non-existence of meromorphic solution of a Fermat type functional equation, Aequationes Math. 76(2008), 140-150], Chen, Chen and Li [Uniqueness of difference operators of meromorphic functions, J. Ineq. Appl. 2012(2012), Art 48].

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1 Introduction, definitions and results

Throughout this paper, a meromorphic function always means meromorphic in the whole complex plane, unless specifically stated otherwise. Let k be a positive integer or infinity and $a \in C \cup \{\infty\}$. Set $E(a, f) = \{z : f(z) - a = 0\}$, where a zero point with multiplicity k is counted k times in the set. If these zeros points are only counted once, then we denote the set by $\overline{E}(a, f)$. Let f and g be two nonconstant meromorphic functions. If $E(a, f) = E(a, g)$, then we say that f and g share the value a CM; if $\overline{E}(a, f) = \overline{E}(a, g)$, then we say that f and g share the value a IM. We denote by $E_{k)}(a, f)$ the set of all a-points of f with multiplicities not exceeding k, where an a-point is counted according to its multiplicity. Also we denote by $\overline{E}_{k)}(a, f)$ the set of distinct a-points of f with multiplicities not greater than k. It is assumed that the reader is familiar with the notations of Nevanlinna theory such as $T(r, f)$, $m(r, f)$, $N(r, f)$, $\overline{N}(r, f)$, $S(r, f)$ and so on, that can be found, for instance, in [12][19]. We denote by $N_{k)}(r, 1/(f - a))$ the counting function for zeros of $f - a$ with multiplicity less than or equal to k, and by $\overline{N}_{k)}(r, 1/(f - a))$ the corresponding one for which multiplicity is not counted. Let $N_{(k}(r, 1/(f - a))$ be the counting function for zeros of $f - a$ with multiplicity at least k and $\overline{N}_{(k}(r, 1/(f - a))$ the corresponding one for which multiplicity is not counted. In the following we are recalling the notion.

Definition 1.1. [14] For a complex number $a \in C \cup \{\infty\}$, we denote by $E_k(a, f)$ the set of all a-points of f where an a-point with mutiplicity m is counted m times if $m \leq k$ and $k + 1$ times if $m > k$. For a complex number $a \in C \cup \{\infty\}$, such that $E_k(a, f) = E_k(a, g)$, then we say that f and g share the value a with weight k.

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The definition implies that if f, g share a value a with weight k, then z_0 is a zero of $f-a$ with multiplicity $m(\leq k)$ if and only if it is a zero of $g-a$ with multiplicity $m(\leq k)$ and z_0 is a zero of $f-a$ with multiplicity $m(>k)$ if and only if it is a zero of $g-a$ with multiplicity $n(>k)$, where m is not necessarily equal to n. We write f, g share (a,k) to mean that f, g share the value a with weight k. Clearly if f, g share (a,k) then f, g share (a,p) for all integer p, $0 \leq p < k$. Also we note that f, g share a value a IM or CM if and only if f, g share $(a,0)$ or (a,∞) respectively.

In 1996, R. Brück posed the following conjecture.

Conjecture [4]. Let f be a nonconstant entire function. Suppose that $\rho_1(f)$ is not a positive integer or infinite, if f and f' share one finite value a CM, then

$$\frac{f'-a}{f-a} = c\,,$$

for some non-zero constant c, where $\rho_1(f)$ is the first iterated order of f which is defined by

$$\rho_1(f) = \limsup_{r\to\infty} \frac{\log\log T(r,f)}{\log r}\,.$$

In 1998, Gundersen and Yang [10] proved that the conjecture is true if f is of finite order, and in 1999, Yang [21] generalized their result to the k-th derivatives. In 2004, Chen and Shon [7] proved that the conjecture is true for entire functions of first iterated order $\rho_1(f) < 1/2$. In 2008, Yang and Zhang considered the uniqueness problems on meromorphic function f^n sharing value with its first derivative. One of their results can be stated as follows.

Theorem 1.2. *[22] Let $f(z)$ be a non-constant meromorphic function and $n \geq 12$ be an integer. Let $F = f^n$. If F and F' share 1 CM, then $F = F'$, and f assumes the form $f(z) = ce^{\frac{1}{n}z}$.*

Recently the difference Nevanlinna theory and its applications to the uniqueness theory has becomes a subject of increasing interest among the researchers. Readers can make a glance on the references [2][3][5][8][11][13][15][23]. In 2012, Chen, Chen and Li replaced F' by $\Delta_c F$ where $\Delta_c F = F(z+c) - F(z)$ and proved the following theorem.

Theorem 1.3. *[6] Let $f(z)$ be a non-constant meromorphic function of finite order and $n \geq 9$ be an integer. Let $F(z) = f(z)^n$. If $F(z)$ and $\Delta_c F$ share 1, ∞ CM, then $F(z) = \Delta_c F$.*

It is natural to ask whether F' can be replaced by the q-shift $F(qz+\eta)$ in Theorem 1.2. Considering this question, we prove the following results.

Theorem 1.4. *Let f be a non-constant meromorphic function of zero-order. Suppose that q is a non-zero complex constant, $\eta \in C$ and n is an integer satisfying $n \geq 7$. If $f^n(z)$ and $f^n(qz+\eta)$ share $(1,2)$, $f(z)$ and $f(qz+\eta)$ share (∞,∞), then $f(z) = tf(qz+\eta)$, where t is a constant and $t^n = 1$.*

Corollary 1.5. *Let f be a non-constant entire function of zero-order. Suppose that q is a non-zero complex constant, $\eta \in C$ and n is an integer satisfying $n \geq 5$. If $f^n(z)$ and $f^n(qz+\eta)$ share $(1,2)$, then $f(z) = tf(qz+\eta)$, where t is a constant and $t^n = 1$.*

Theorem 1.6. *Let f be a non-constant meromorphic function of zero-order. Suppose that q is a non-zero complex constant, $\eta \in C$ and n is an integer satisfying $n \geq 8$. If $f^n(z)$ and $f^n(qz+\eta)$ share $(1,2)$, $f(z)$ and $f(qz+\eta)$ share $(\infty,0)$, then $f(z) = tf(qz+\eta)$, where t is a constant and $t^n = 1$.*

Theorem 1.7. *Let f be a non-constant meromorphic function of zero-order. Suppose that q is a non-zero complex constant, $\eta \in C$ and n is an integer satisfying $n \geq 7$. If $f(z)$ and $f(qz+\eta)$ share (∞,∞) and $E_{3)}(1, f^n(z)) = E_{3)}(1, f^n(qz+\eta))$, then $f(z) = tf(qz+\eta)$, where t is a constant and $t^n = 1$.*

Theorem 1.8. *Let f be a non-constant meromorphic function of zero-order. Suppose that q is a non-zero complex constant, $\eta \in C$ and n is an integer satisfying $n \geq 8$. If $f(z)$ and $f(qz+\eta)$ share $(\infty,0)$ and $E_{3)}(1, f^n(z)) = E_{3)}(1, f^n(qz+\eta))$, then $f(z) = tf(qz+\eta)$, where t is a constant and $t^n = 1$.*

Remark 1.1. If we set $S_1 = \{1, \omega, ..., \omega^{n-1}\}$, $S_2 = \{\infty\}$, we can get the following equivalent forms.

Theorem 1.9. *Let S_1, S_2 be given as above. Suppose f is a non-constant zero-order meromorphic function such that*

$$E_{f(z)}(S_1, 2) = E_{f(qz+\eta)}(S_1, 2)\,, E_{f(z)}(S_2, \infty) = E_{f(qz+\eta)}(S_2, \infty)\,.$$

If $n \geq 7$, then $f(z) = tf(qz+\eta)$, where t is a constant and $t^n = 1$.

Corollary 1.10. *Let S_1 be given as above. Suppose f is a non-constant zero-order entire function such that*

$$E_{f(z)}(S_1, 2) = E_{f(qz+\eta)}(S_1, 2)\,.$$

If $n \geq 5$, then $f(z) = tf(qz+\eta)$, where t is a constant and $t^n = 1$.

Theorem 1.11. *Let S_1, S_2 be given as above. Suppose f is a non-constant zero-order meromorphic function such that*

$$E_{f(z)}(S_1, 2) = E_{f(qz+\eta)}(S_1, 2)\,, E_{f(z)}(S_2, 0) = E_{f(qz+\eta)}(S_2, 0)\,.$$

If $n \geq 8$, then $f(z) = tf(qz+\eta)$, where t is a constant and $t^n = 1$.

Theorem 1.12. *Let S_1, S_2 be given as above. Suppose f is a non-constant zero-order meromorphic function such that*

$$E_{3)}(S_1, f(z)) = E_{3)}(S_1, f(qz+\eta))\,, E_{f(z)}(S_2, \infty) = E_{f(qz+\eta)}(S_2, \infty)\,.$$

If $n \geq 7$, then $f(z) = tf(qz+\eta)$, where t is a constant and $t^n = 1$.

Theorem 1.13. *Let S_1, S_2 be given as above. Suppose f is a non-constant zero-order meromorphic function such that*

$$E_{3)}(S_1, f(z)) = E_{3)}(S_1, f(qz+\eta))\,, E_{f(z)}(S_2, 0) = E_{f(qz+\eta)}(S_2, 0)\,.$$

If $n \geq 8$, then $f(z) = tf(qz+\eta)$, where t is a constant and $t^n = 1$.

2 Some Lemmas

In this section, we present some lemmas which will be needed in the sequel. We will denote by H the following function:

$$H = \left(\frac{F''}{F'} - \frac{2F'}{F-1}\right) - \left(\frac{G''}{G'} - \frac{2G'}{G-1}\right).$$

Lemma 2.1. *[18] Let f be a non-constant meromorphic function, and let $a_1, a_2, ..., a_n$ be finite complex numbers, $a_n \neq 0$. Then*

$$T(r, a_n f^n + \cdots + a_2 f^2 + a_1 f + a_0) = nT(r, f) + S(r, f).$$

Lemma 2.2. *[1] Let F, G be two non-constant meromorphic functions. If F, G share $(1, 2)$ and (∞, k), where $0 \leq k \leq \infty$. If $H \not\equiv 0$, then*

$$T(r, F) \leq N_2\left(r, \frac{1}{F}\right) + N_2\left(r, \frac{1}{G}\right) + \overline{N}(r, F) + \overline{N}(r, G) + \overline{N}_*(r, \infty; F, G) + S(r, F) + S(r, G),$$

where $\overline{N}_*(r, \infty; F, G)$ denotes the reduced counting function of those a-points of F whose multiplicities differ from the multiplicities of the corresponding a-points of G.

Lemma 2.3. *[16] [17] Let $f(z)$ be a non-constant zero-order meromorphic function. Suppose that q is a non-zero complex constant, $\eta \in C$. Then*

$$T(r, f(qz + \eta)) = T(r, f(z)) + S_1(r, f),$$

where $S_1(r, f) = o(T(r, f))$ for all r on a set of logarithmic density 1.

Lemma 2.4. *[1] Let F, G be two nonconstant meromorphic functions. If F, G share (∞, k) and $E_{3)}(1, F) = E_{3)}(1, G)$, where $0 \leq k \leq \infty$. If $H \not\equiv 0$, then*

$$T(r, F) + T(r, G) \leq 2N_2\left(r, \frac{1}{F}\right) + 2N_2\left(r, \frac{1}{G}\right) + 2\overline{N}(r, F) + 2\overline{N}(r, G) + 2\overline{N}_*(r, \infty; F, G) + S(r, F) + S(r, G),$$

where $\overline{N}_*(r, \infty; F, G)$ denotes the reduced counting function of those a-points of F whose multiplicities differ from the multiplicities of the corresponding a-points of G.

3 Proof of Theorem 1.4

Let

$$F = f^n(z), G = f^n(qz + \eta). \tag{1}$$

Then it is easy to verify F and G share $(1, 2)$ and (∞, ∞). Let H be defined as above. Suppose that $H \not\equiv 0$. It follows from Lemma 2.2 that

$$T(r, F) \leq N_2\left(r, \frac{1}{F}\right) + N_2\left(r, \frac{1}{G}\right) + \overline{N}(r, F) + \overline{N}(r, G) + \overline{N}_*(r, \infty; F, G) + S(r, F) + S(r, G). \tag{2}$$

According to Lemma 2.1, we have

$$T(r,F) = nT(r,f) + S(r,f)\,. \tag{3}$$

It's obvious that

$$N_2\left(r,\frac{1}{F}\right) = N_2\left(r,\frac{1}{f^n(z)}\right) = 2\overline{N}\left(r,\frac{1}{f}\right)\,, \tag{4}$$

$$N_2\left(r,\frac{1}{G}\right) = N_2\left(r,\frac{1}{f^n(qz+\eta)}\right) = 2\overline{N}\left(r,\frac{1}{f(qz+\eta)}\right)\,, \tag{5}$$

$$\overline{N}(r,F) = \overline{N}(r,G) = \overline{N}(r,f)\,, \tag{6}$$

$$\overline{N}_*(r,\infty;F,G) = 0\,. \tag{7}$$

Combining (2), (3), (4), (5),(6) and (7), we deduce

$$\begin{aligned} T(r,F) = nT(r,f) + S(r,f) &\le 2\overline{N}\left(r,\frac{1}{f}\right) + 2\overline{N}\left(r,\frac{1}{f(qz+\eta)}\right) \\ &+2\overline{N}(r,f) + S(r,f) \le 6T(r,f) + S_1(r,f)\,, \end{aligned} \tag{8}$$

that is,

$$(n-6)T(r,f) \le S_1(r,f)\,, \tag{9}$$

which contradicts with $n \ge 7$. Therefore $H \equiv 0$. By integration, we get

$$\frac{1}{F-1} = \frac{A}{G-1} + B\,, \tag{10}$$

where $A \neq 0$ and B are constants. From (10) we have

$$G = \frac{(B-A)F + (A-B-1)}{BF - (B+1)}\,. \tag{11}$$

We discuss the following three cases.

Case I. Suppose that $B \neq 0, -1$. From (11), we have

$$\overline{N}\left(r,\frac{1}{F-\frac{B+1}{B}}\right) = \overline{N}(r,G)\,. \tag{12}$$

From the second fundamental theorem, we have

$$\begin{aligned} nT(r,f) = T(r,F) + S(r,f) &\le \overline{N}(r,F) + \overline{N}\left(r.\frac{1}{F}\right) \\ &+\overline{N}\left(r,\frac{1}{F-\frac{B+1}{B}}\right) + S(r,f) \\ &\le 2\overline{N}(r,f) + \overline{N}\left(r,\frac{1}{f}\right) + S(r,f)\,, \end{aligned} \tag{13}$$

which contradicts with $n \geq 7$.

Case II. Suppose that $B = 0$. From (11), we have

$$G = AF - (A-1)\,. \tag{14}$$

If $A \neq 1$, from (14) we obtain

$$\overline{N}\left(r, \frac{1}{F - \frac{A-1}{A}}\right) = \overline{N}\left(r, \frac{1}{G}\right)\,. \tag{15}$$

From the second fundamental theorem and Lemma 2.1, we have

$$\begin{aligned} nT(r,f) = T(r,F) + S(r,f) &\leq \overline{N}(r,F) + \overline{N}\left(r.\frac{1}{F}\right) \\ &\quad + \overline{N}\left(r, \frac{1}{F - \frac{A-1}{A}}\right) + S(r,f) \\ &\leq \overline{N}(r,f) + \overline{N}\left(r, \frac{1}{f}\right) + \overline{N}\left(r, \frac{1}{f(qz+\eta)}\right) + S(r,f)\,, \end{aligned} \tag{16}$$

which contradicts with $n \geq 7$. Thus $A = 1$. From (14) we have $F = G$, that is $f^n(z) = f^n(qz+\eta)$. So we deduce $f(z) = tf(qz+\eta)$, where t is a constant and $t^n = 1$.

Case III. Suppose that $B = -1$. From (11) we have

$$G = \frac{(A+1)F - A}{F}\,. \tag{17}$$

If $A \neq -1$, we obtain from (17) that

$$\overline{N}\left(r, \frac{1}{F - \frac{A}{A+1}}\right) = \overline{N}\left(r, \frac{1}{G}\right)\,. \tag{18}$$

By the same reasoning discussed in Case II, we obtain a contradiction. Hence $A = -1$. From (17), we get $FG = 1$, that is

$$f^n(z) f^n(qz+\eta) = 1\,. \tag{19}$$

From (19) we obtain $f(z) \neq 0, \infty$, then $f(z) = e^{h(z)}$, where $h(z)$ is an entire function. Since $f(z)$ is of zero order, we can deduce $f(z)$ is a constant, which is a contradiction. This completes the proof of Theorem 1.4.

4 Proof of Theorem 1.6

Let

$$F = f^n(z), G = f^n(qz+\eta)\,. \tag{20}$$

Then it is easy to verify F and G share $(1,2)$ and $(\infty,0)$. Let H be defined as above. Suppose that $H \not\equiv 0$. It follows from Lemma 2.2 that

$$T(r,F) \leq N_2\left(r,\frac{1}{F}\right)+N_2\left(r,\frac{1}{G}\right)+\overline{N}(r,F)+\overline{N}(r,G) \\ +\overline{N}_*(r,\infty;F,G)+S(r,F)+S(r,G)\,. \tag{21}$$

According to Lemma 2.1, we have

$$T(r,F)=nT(r,f)+S(r,f)\,. \tag{22}$$

It's obvious that

$$N_2\left(r,\frac{1}{F}\right)=N_2\left(r,\frac{1}{f^n(z)}\right)=2\overline{N}\left(r,\frac{1}{f}\right)\,, \tag{23}$$

$$N_2\left(r,\frac{1}{G}\right)=N_2\left(r,\frac{1}{f^n(qz+\eta)}\right)=2\overline{N}\left(r,\frac{1}{f(qz+\eta)}\right)\,, \tag{24}$$

$$\overline{N}(r,F)=\overline{N}(r,G)=\overline{N}(r,f)\,, \tag{25}$$

$$\overline{N}_*(r,\infty;F,G)\leq\overline{N}(r,f)\,. \tag{26}$$

Combining (21), (22), (23), (24),(25) and (26), we deduce

$$T(r,F)=nT(r,f)+S(r,f)\leq 2\overline{N}\left(r,\frac{1}{f}\right)+2\overline{N}\left(r,\frac{1}{f(qz+\eta)}\right) \\ +3\overline{N}(r,f)+S(r,f)\leq 7T(r,f)+S_1(r,f)\,, \tag{27}$$

that is,

$$(n-7)T(r,f)\leq S_1(r,f)\,, \tag{28}$$

which contradicts with $n\geq 8$. Therefore $H\equiv 0$. Similar to the arguments in Theorem 1.4, we see that Theorem 1.6 holds.

5 Proof of Theorem 1.7

Let

$$F=f^n(z),G=f^n(qz+\eta)\,. \tag{29}$$

Then it is easy to verify F and G share $E_{3)}(1,F)=E_{3)}(1,G)$ and (∞,∞). Let H be defined as above. Suppose that $H\not\equiv 0$. It follows from Lemma 2.4 that

$$T(r,F)+T(r,G)\leq 2N_2\left(r,\frac{1}{F}\right)+2N_2\left(r,\frac{1}{G}\right)+2\overline{N}(r,F)+2\overline{N}(r,G) \\ +2\overline{N}_*(r,\infty;F,G)+S(r,F)+S(r,G)\,. \tag{30}$$

According to Lemma 2.1, we have

$$T(r,F)=nT(r,f)+S(r,f)\,. \tag{31}$$

By Lemma 2.1 and Lemma 2.3, we have

$$T(r,G)=nT(r,f(qz+\eta))+S(r,f)=nT(r,f)+S_1(r,f)\,. \tag{32}$$

It's obvious that

$$N_2\left(r,\frac{1}{F}\right)=N_2\left(r,\frac{1}{f^n(z)}\right)=2\overline{N}\left(r,\frac{1}{f}\right), \tag{33}$$

$$N_2\left(r,\frac{1}{G}\right)=N_2\left(r,\frac{1}{f^n(qz+\eta)}\right)=2\overline{N}\left(r,\frac{1}{f(qz+\eta)}\right), \tag{34}$$

$$\overline{N}(r,F)=\overline{N}(r,G)=\overline{N}(r,f)\,, \tag{35}$$

$$\overline{N}_*(r,\infty;F,G)=0\,. \tag{36}$$

Combining (30), (31), (32), (33),(34), (35) and (36), we deduce

$$\begin{aligned} T(r,F)+T(r,G)&=2nT(r,f)+S_1(r,f)\\ &\le 4\overline{N}\left(r,\frac{1}{f}\right)+4\overline{N}\left(r,\frac{1}{f(qz+\eta)}\right)+4\overline{N}(r,f)+S_1(r,f)\\ &\le 12T(r,f)+S_1(r,f)\,, \end{aligned} \tag{37}$$

that is,

$$(2n-12)T(r,f)\le S_1(r,f)\,, \tag{38}$$

which contradicts with $n\ge 7$. Therefore $H\equiv 0$. Similar to the arguments in Theorem 1.4, we see that Theorem 1.7 holds.

6 Proof of Theorem 1.8

Let

$$F=f^n(z), G=f^n(qz+\eta)\,. \tag{39}$$

Then it is easy to verify F and G share $E_{3)}(1,F)=E_{3)}(1,G)$ and $(\infty,0)$. Let H be defined as above. Suppose that $H\not\equiv 0$. It follows from Lemma 2.4 that

$$\begin{aligned} T(r,F)+T(r,G)\le{}& 2N_2\left(r,\frac{1}{F}\right)+2N_2\left(r,\frac{1}{G}\right)+2\overline{N}(r,F)+2\overline{N}(r,G)\\ &+2\overline{N}_*(r,\infty;F,G)+S(r,F)+S(r,G)\,. \end{aligned} \tag{40}$$

According to Lemma 2.1, we have

$$T(r,F) = nT(r,f) + S(r,f)\,. \tag{41}$$

By Lemma 2.1 and Lemma 2.3, we have

$$T(r,G) = nT(r,f(qz+\eta)) + S(r,f) = nT(r,f) + S_1(r,f)\,. \tag{42}$$

It's obvious that

$$N_2\left(r,\frac{1}{F}\right) = N_2\left(r,\frac{1}{f^n(z)}\right) = 2\overline{N}\left(r,\frac{1}{f}\right)\,, \tag{43}$$

$$N_2\left(r,\frac{1}{G}\right) = N_2\left(r,\frac{1}{f^n(qz+\eta)}\right) = 2\overline{N}\left(r,\frac{1}{f(qz+\eta)}\right)\,, \tag{44}$$

$$\overline{N}(r,F) = \overline{N}(r,G) = \overline{N}(r,f)\,, \tag{45}$$

$$\overline{N}_*(r,\infty;F,G) \le \overline{N}(r,f)\,. \tag{46}$$

Combining (40), (41), (42), (43), (44), (45) and (46), we deduce

$$\begin{aligned} T(r,F) + T(r,G) &= 2nT(r,f) + S_1(r,f) \\ &\le 4\overline{N}\left(r,\frac{1}{f}\right) + 4\overline{N}\left(r,\frac{1}{f(qz+\eta)}\right) + 6\overline{N}(r,f) + S_1(r,f) \\ &\le 14T(r,f) + S_1(r,f)\,, \end{aligned} \tag{47}$$

that is,

$$(2n-14)T(r,f) \le S_1(r,f)\,, \tag{48}$$

which contradicts with $n \ge 8$. Therefore $H \equiv 0$. Similar to the arguments in Theorem 1.4, we see that Theorem 1.8 holds.

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