Connection and duplication formulas for the Boas-Buck-Appell polynomials

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Abstract

The present paper conduct to introduce the connection and duplication formulas associated with the Boas-Buck-Appell polynomials. Examples providing the analogues results for certain members related to the Boas-Buck-Appell polynomials are considered.

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1 Introduction and preliminaries

Due to their marvelous applications in numerous fields, the Appell polynomials [3] construct an indispensable class of polynomials. The Appell polynomials come out in different applications in pure and applied mathematics. These polynomials arise in theoretical physics, chemistry [4, 15] and several branches of mathematics [11] such as the study of polynomial expansions of analytic functions, number theory and numerical analysis.

The Appell polynomials [3] may be defined by either of the following equivalent conditions:

$\{A_n(x)\}$ $(n \in \mathbb{N}_0)$ is an Appell set ($A_n(x)$ being of degree exactly n), if either

(i) $\frac{d}{dx}A_n(x) = n\ A_{n-1}(x), \quad n \in \mathbb{N}_0$, or

(ii) there exists an exponential generating function of the form

$$\mathcal{G}(t)\exp(xt) = \sum_{n=0}^{\infty} A_n(x)\frac{t^n}{n!}, \tag{1.1}$$

where $\mathcal{G}(t)$ has (at least the formal) expansion:

$$\mathcal{G}(t) = \sum_{n=0}^{\infty} A_n\frac{t^n}{n!}, \quad A_0 \neq 0. \tag{1.2}$$

Members of the Appell family can be obtained by selecting the function $\mathcal{G}(t)$ in generating function (1.1). Some of these members are listed in Table 1.

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Table 1. Certain members belonging to the Appell family

S. No.	$\mathcal{G}(t)$	Generating function	Name of the polynomials
I.	$\frac{t}{e^t-1}$	$\frac{t}{e^t-1}\ e^{xt} = \sum_{n=0}^{\infty} B_n(x)\frac{t^n}{n!}$	The Bernoulli polynomials [13]
II.	$\frac{2}{e^t+1}$	$\frac{2}{e^t+1}e^{xt} = \sum_{n=0}^{\infty} E_n(x)\frac{t^n}{n!}$	The Euler polynomials [13]
III.	$\frac{2t}{(e^t+1)}$	$\frac{2t}{(e^t+1)}e^{xt} = \sum_{n=0}^{\infty} G_n(x)\frac{t^n}{n!}$	The Genocchi polynomials [9]
IV.	$e^{\frac{-\nu t^2}{2}}$	$e^{-\frac{\nu t^2}{2}}e^{xt} = \sum_{n=0}^{\infty} H_k^{(\nu)}(x)\frac{t^n}{n!}$	The Hermite polynomials of variance ν [12]

The notions of the monomiality principle and operational techniques are used to unify the special polynomials and to obtain hybrid forms of the special polynomials. Recently, the Boas-Buck-Appell polynomials ${}_pA_n(x)$ are introduced which are defined by means of the following generating function [1]:

$$\mathcal{G}(t)A(t)\psi(xH(t)) = \sum_{n=0}^{\infty} {}_pA_n(x)\frac{t^n}{n!}, \tag{1.3}$$

where $\mathcal{G}(t)$ has same expansion as in equation (1.2) and $A(t)$, $\psi(t)$, $H(t)$ are power series such that

$$A(t) = \sum_{n=0}^{\infty} a_n\frac{t^n}{n!},\ a_0 \neq 0; \tag{1.4}$$

$$\psi(t) = \sum_{n=0}^{\infty} \gamma_n\frac{t^n}{n!},\ \gamma_n \neq 0,\ \forall\ n, \tag{1.5}$$

with $\psi(t)$ not a polynomial and

$$H(t) = \sum_{n=0}^{\infty} h_n\frac{t^n}{n!},\ h_0 \neq 0. \tag{1.6}$$

Remark 1.1. Taking $H(t) = t$ in equation (1.3), the Boas-Buck-Appell polynomials ${}_pA_n(x)$ reduce to the Brenke-Appell polynomials ${}_YA_n(x)$. The Brenke-Appell polynomials ${}_YA_n(x)$ possess the following generating equation [1]:

$$\mathcal{G}(t)A(t)\psi(xt) = \sum_{n=0}^{\infty} {}_YA_n(x)\frac{t^n}{n!}, \tag{1.7}$$

where $\mathcal{G}(t), A(t)$ and $\psi(t)$ has same expansion as given by equations (1.2), (1.4) and (1.5).

Remark 1.2. For $\mathcal{G}(t) = 1$, generating relation (1.3) gives generating function for the Boas-Buck polynomials [5]:

$$A(t)\psi(xH(t)) = \sum_{n=0}^{\infty} p_n(x)\frac{t^n}{n!}, \tag{1.8}$$

where $A(t)$, $\psi(t)$ and $H(t)$ have same expansions as defined above.

Members of the Boas-Buck family can be obtained by selecting the appropriate functions $A(t)$ and $H(t)$ in generating equation (1.8). Some of these members are listed in Table 2.

Table 2. Certain members belonging to the Boas-Buck family

S. No.	$A(t)$	Generating function	Name of the polynomials
I.	e^t	$e^t\, {}_pF_q\left(\begin{matrix}(a)_p\\(b)_q\end{matrix}\Big\vert -xt\right) = \sum\limits_{n=0}^{\infty} \mathfrak{B}_n\left((a)_p,(b)_q;x\right)\frac{t^n}{n!}$	The Brafman polynomials [6, 7]
II.	$(1-t)^{-\lambda}$	$(1-t)^{-\lambda}\, {}_pF_q\left(\begin{matrix}(a)_p\\(b)_q\end{matrix}\Big\vert xt\right) = \sum\limits_{n=0}^{\infty} \frac{(\lambda)_n}{n!} P_n^{\lambda}(x)\, t^n$	The Chaunday polynomials [7]
III.	$(1-pt)^N$	$(1-pt)^N \left(\frac{1+qt}{1-pt}\right)^x = \sum\limits_{n=0}^{\infty} \tilde{K}_n^p(x,N)\frac{t^n}{n!}$	The Krawtchouk polynomials [14]

In last few years, studies of the connection and duplication problems have gained growing interest. Very recently, Subuhi Khan and T. Nahid [10] obtained the connection and duplication formulas for the hybrid family of special polynomials. The computation of the connection and duplication coefficients perform an essential role in various situations of pure and applied mathematics, especially in combinatorial analysis. The connection and duplication problems are not only important from a fundamental point of view but also because they are used in computation of physical and chemical properties of quantum-mechanical systems.

Given two polynomial sets $\{P_n\}_{n\geq 0}$ and $\{Q_n\}_{n\geq 0}$, the connection problem between them is to find the coefficients $C_m(n)$ in the expression:

$$Q_n(x) = \sum_{m=0}^{n} \mathcal{C}_m(n) P_m(x). \tag{1.9}$$

A special case of the connection problem, called the duplication problem consists in finding the coefficients $C_m(n,\alpha)$ in the expansion:

$$P_n(\alpha x) = \sum_{m=0}^{n} \mathcal{C}_m(n,\alpha) P_m(x), \tag{1.10}$$

where α designates a non-zero complex number.

The current popularity and growing interest in detecting connection and duplication coefficients is the applications of such kind of problems in several branches of mathematics and physics. The main goal of the present paper is to give an alternative method for finding the connection and duplication problems for the Boas-Buck-Appell polynomials. Examples providing the analogous results for certain members related to the Boas-Buck-Appell polynomials have been constructed.

2 Main results

In this section, the connection and duplication formulas for the Boas-Buck-Appell polynomials ${}_pA_n(x)$ is obtained by proving the following results.

Theorem 2.1. Let $\{{}_pA_n^{(1)}(x)\}_{n\geq 0}$ and $\{{}_pA_n^{(2)}(x)\}_{n\geq 0}$ be two Boas-Buck-Appell polynomials sets, which are specified by the following generating functions:

$$\mathcal{G}_1(t)A_1(t)\psi(xH_1(t)) = \sum_{n=0}^{\infty} {}_pA_n^{(1)}(x)\frac{t^n}{n!} \tag{2.1}$$

and

$$\mathcal{G}_2(t)A_2(t)\psi(xH_2(t)) = \sum_{n=0}^{\infty} {}_pA_n^{(2)}(x)\frac{t^n}{n!}, \tag{2.2}$$

respectively. Then the following connection formula holds true:

$${}_pA_n^{(2)}(x) = \sum_{m=0}^{n} \mathcal{C}_m(n)\ {}_pA_m^{(1)}(x), \tag{2.3}$$

where $\mathcal{C}_m(n) = \sum\limits_{r=0}^{m} \frac{a_{n-r}n!}{r!}C_m(r)$.

Proof. Generating function (2.2) can be written as

$$\frac{\mathcal{G}_2(t)A_2(t)}{\mathcal{G}_1(\varphi(t))A_1(\varphi(t))}\ \mathcal{G}_1(\varphi(t))A_1(\varphi(t))\psi(xH_1(\varphi(t))) = \sum_{n=0}^{\infty} {}_pA_n^{(2)}(x)\frac{t^n}{n!}, \tag{2.4}$$

where $\varphi(t) = H_1^{-1}(H_2(t))$. Letting $\frac{\mathcal{G}_2(t)}{\mathcal{G}_1(\varphi(t))} = \sum\limits_{n=0}^{\infty} a_n t^n$ and using equation (2.1) and result [8, p.539, Lemma 2.3] in equation (2.4), it follows that

$$\sum_{n=0}^{\infty} a_n t^n \sum_{k=m}^{\infty} \frac{m!}{k!}C_m(k)t^k \sum_{m=0}^{\infty} \frac{{}_pA_m^{(1)}(x)}{m!} = \sum_{n=0}^{\infty} {}_pA_n^{(2)}(x)\frac{t^n}{n!}, \tag{2.5}$$

which on replacing k by r and then n by $n-r$ becomes

$$\sum_{n=0}^{\infty}\sum_{r=0}^{n}\sum_{m=0}^{r} \frac{a_{n-r}C_m(r)t^n}{r!}{}_pA_m^{(1)}(x) = \sum_{n=0}^{\infty} {}_pA_n^{(2)}(x)\frac{t^n}{n!}. \tag{2.6}$$

Equating the coefficients of identical powers of t on both sides of equation (2.6), assertion (2.3) follows.

Q.E.D.

Further, the duplication formula involving the Boas-Buck-Appell polynomials ${}_pA_n(x)$ is obtained in the form of following result:

Theorem 2.2. The Boas-Buck-Appell polynomial set $\{{}_pA_n(x)\}_{n\geq 0}$ possess the following duplication formula:

$${}_pA_n(\alpha x) = \sum_{m=0}^{n} \mathcal{C}_m(n,\alpha)\ {}_pA_m(x), \tag{2.7}$$

where $\mathcal{C}_m(n,\alpha) = \sum_{k=0}^{n-m} \binom{n}{m}\binom{n-m}{k}\alpha^m \beta_{n-m-k}(\alpha)\gamma_k(\alpha)$ and
$\mathrm{g(t)} = \frac{\mathcal{G}(t)A(t)}{\mathcal{G}(\alpha t)A(\alpha t)} = \sum_{n,k=0}^{\infty} \beta_n(\alpha)\gamma_k(\alpha)\frac{t^{n+k}}{n!\ k!}$.

Proof. Generating function (1.3) can be written as

$$\frac{\mathcal{G}(t)A(t)}{\mathcal{G}(\alpha t)A(\alpha t)}\ \mathcal{G}(\alpha t)A(\alpha t)\psi(\alpha x H(t)) = \sum_{n=0}^{\infty} {}_pA_n(\alpha x)\frac{t^n}{n!}. \tag{2.8}$$

Let

$$g(t) = \frac{\mathcal{G}(t)A(t)}{\mathcal{G}(\alpha t)A(\alpha t)} = \sum_{n,k=0}^{\infty} \beta_n(\alpha)\gamma_k(\alpha)\frac{t^{n+k}}{n!\ k!},$$

which on applying the Cauchy-product rule becomes

$$g(t) = \frac{\mathcal{G}(t)A(t)}{\mathcal{G}(\alpha t)A(\alpha t)} = \sum_{n=0}^{\infty}\sum_{k=0}^{n} \binom{n}{k}\beta_{n-k}(\alpha)\gamma_k(\alpha)\frac{t^n}{n!}. \tag{2.9}$$

In view of equations (1.3), (2.8) and (2.9), it follows that

$$\sum_{n=0}^{\infty}\sum_{k=0}^{n} \binom{n}{k}\beta_{n-k}(\alpha)\gamma_k(\alpha)\frac{t^n}{n!}\sum_{m=0}^{\infty} {}_pA_m(x)\frac{\alpha^m t^m}{m!} = \sum_{n=0}^{\infty} {}_pA_n(\alpha x)\frac{t^n}{n!}. \tag{2.10}$$

Again, applying the Cauchy-product rule in the l.h.s. of equation (2.10) and then equating the coefficients of identical powers of t in the obtained relation, we get assertion (2.7). Q.E.D.

In the next section, connection and duplication formulas for certain members related to the Boas-Buck-Appell polynomials are established.

3 Applications

In this section, certain examples are constructed in order to give the applications of the results obtained in the previous section.

First, we give certain members belonging to Boas-Buck-Appell family by selecting suitable function $\mathcal{G}(t)$ and $A(t)$. These members are mentioned in Table 3.

Table 3. Certain members belonging to the Boas-Buck-Appell family

S. No.	$\mathcal{G}(t);\ A(t)$	Generating function	Name of the polynomials
I.	$\frac{t}{e^t-1};\ e^t$	$\frac{te^t}{e^t-1}\,{}_pF_q\left(\begin{matrix}(a)_p\\(b)_q\end{matrix}\middle\vert -xt\right)=\sum_{n=0}^{\infty}{}_{\mathfrak{B}}B_n\left((a)_p,(b)_q;x\right)\frac{t^n}{n!}$	The Brafman-Bernoulli polynomials
II.	$\frac{2}{e^t+1};\ e^t$	$\frac{2e^t}{e^t+1}\,{}_pF_q\left(\begin{matrix}(a)_p\\(b)_q\end{matrix}\middle\vert -xt\right)=\sum_{n=0}^{\infty}{}_{\mathfrak{B}}E_n\left((a)_p,(b)_q;x\right)\frac{t^n}{n!}$	The Brafman-Euler polynomials
III.	$e^{-\nu t^2/2};\ e^t$	$e^{t-\nu t^2/2}\,{}_pF_q\left(\begin{matrix}(a)_p\\(b)_q\end{matrix}\middle\vert -xt\right)=\sum_{n=0}^{\infty}{}_{\mathfrak{B}}H_n^{\nu}\left((a)_p,(b)_q;x\right)\frac{t^n}{n!}$	The Brafman-Hermite polynomials
IV.	$\frac{t}{e^t-1};\ (1-t)^{-\lambda}$	$\frac{t(1-t)^{-\lambda}}{e^t-1}\,{}_pF_q\left(\begin{matrix}(a)_p\\(b)_q\end{matrix}\middle\vert xt\right)=\sum_{n=0}^{\infty}\frac{(\lambda)_n}{n!}\,{}_PB_n^{\lambda}(x)\,t^n$	The Chaunday-Bernoulli polynomials
V.	$\frac{2}{e^t+1};\ (1-t)^{-\lambda}$	$\frac{2(1-t)^{-\lambda}}{e^t+1}\,{}_pF_q\left(\begin{matrix}(a)_p\\(b)_q\end{matrix}\middle\vert xt\right)=\sum_{n=0}^{\infty}\frac{(\lambda)_n}{n!}\,{}_PE_n^{\lambda}(x)\,t^n$	The Chaunday-Euler polynomials
VI.	$\frac{2t}{e^t+1};\ (1-t)^{-\lambda}$	$\frac{2t(1-t)^{-\lambda}}{e^t+1}\,{}_pF_q\left(\begin{matrix}(a)_p\\(b)_q\end{matrix}\middle\vert xt\right)=\sum_{n=0}^{\infty}\frac{(\lambda)_n}{n!}\,{}_PG_n^{\lambda}(x)\,t^n$	The Chaunday-Genocchi polynomials
VII.	$e^{-\nu t^2/2};\ (1-t)^{-\lambda}$	$e^{-\nu t^2/2}(1-t)^{-\lambda}\,{}_pF_q\left(\begin{matrix}(a)_p\\(b)_q\end{matrix}\middle\vert xt\right)=\sum_{n=0}^{\infty}\frac{(\lambda)_n}{n!}\,{}_PH_n^{\lambda,\nu}(x)\,t^n$	The Chaunday-Hermite polynomials
VIII.	$\frac{t}{e^t-1};\ (1-pt)^N$	$\frac{t(1-pt)^N}{e^t-1}\left(\frac{1+qt}{1-pt}\right)^x=\sum_{n=0}^{\infty}\tilde{K}\,B_n^p(x,N)\frac{t^n}{n!}$	The Krawtchouk-Bernoulli polynomials
IX.	$\frac{2}{e^t+1};\ (1-pt)^N$	$\frac{2(1-pt)^N}{e^t+1}\left(\frac{1+qt}{1-pt}\right)^x=\sum_{n=0}^{\infty}\tilde{K}\,E_n^p(x,N)\frac{t^n}{n!}$	The Krawtchouk-Euler polynomials
X.	$\frac{2t}{e^t+1};\ (1-pt)^N$	$\frac{2t(1-pt)^N}{e^t+1}\left(\frac{1+qt}{1-pt}\right)^x=\sum_{n=0}^{\infty}\tilde{K}\,G_n^p(x,N)\frac{t^n}{n!}$	The Krawtchouk-Genocchi polynomials
XI.	$e^{-\nu t^2/2};\ (1-pt)^N$	$e^{-\nu t^2/2}(1-pt)^N\left(\frac{1+qt}{1-pt}\right)^x=\sum_{n=0}^{\infty}\tilde{K}\,H_n^{p,\nu}(x,N)\frac{t^n}{n!}$	The Krawtchouk-Hermite polynomials

In view of Theorem 2.1, the connection formulas for certain members belonging to the Boas-Buck-Appell family mentioned in Table 3 are established. We present these formulas in Table 4.

Table 4. Connection formulas for certain members belonging to the Boas-Buck-Appell family

S. No.	${}_pA_m^{(1)}(x)$; ${}_pA_n^{(2)}(x)$	Connection formulas
I.	${}_{\mathfrak{B}}H_m^{\nu}((a)_p,(b)_q;x)$; ${}_{\mathfrak{B}}B_n((a)_p,(b)_q;x)$	${}_{\mathfrak{B}}B_n((a)_p,(b)_q;x)=\sum_{m=0}^{n}\sum_{k=0}^{\left[\frac{n-m}{2}\right]}\binom{n}{k}\frac{B_{n-m-2k}\,\nu^k(n-k)!}{2^k m!\,(n-m-2k)!}\,{}_{\mathfrak{B}}H_m^{\nu}((a)_p,(b)_q;x)$
II.	${}_{\mathfrak{B}}B_m((a)_p,(b)_q;x)$; ${}_{\mathfrak{B}}E_n((a)_p,(b)_q;x)$	${}_{\mathfrak{B}}E_n((a)_p,(b)_q;x)=\sum_{m=0}^{n}\sum_{r=0}^{n-m}\binom{n}{m}\binom{n-m}{r}\frac{E_{n-m-r}}{r+1}\,{}_{\mathfrak{B}}B_m((a)_p,(b)_q;x)$
III.	${}_PB_m^{\lambda}(x)$; ${}_PE_n^{\lambda}(x)$	${}_PE_n^{\lambda}(x)=\sum_{m=0}^{n}\sum_{r,k=0}^{r+k\le n-m}\binom{n}{m}\frac{E_{n-m-r-k}\,(\mu-\lambda)_k(\lambda)_m(n-m)!}{(n-m-r-k)!\,k!\,(r+1)!\,(\mu)_n}\,{}_PB_m^{\lambda}(x)$
IV.	${}_PH_m^{\lambda,\nu}(x)$; ${}_PE_n^{\lambda}(x)$	${}_PE_n^{\lambda}(x)=\sum_{m=0}^{n}\sum_{r,k=0}^{k+2r\le n-m}\binom{n}{m}\frac{E_{n-m-k-2r}\,(\mu-\lambda)_k(\lambda)_m\nu^r(n-m)!}{2^r(n-m-k-2r)!\,k!\,r!\,(\mu)_n}\,{}_PH_m^{\lambda,\nu}(x)$
V.	${}_PH_m^{\lambda,\nu}(x)$; ${}_PG_n^{\lambda}(x)$	${}_PG_n^{\lambda}(x)=\sum_{m=0}^{n}\sum_{r=0}^{\left[\frac{n-m}{2}\right]}\binom{n}{m}\frac{G_{n-m-2r}\,(\lambda)_m\nu^r(n-m)!}{2^r(n-m-2r)!\,r!\,(\lambda)_n}\,{}_PH_m^{\lambda,\nu}(x)$
VI.	$\tilde{K}B_m^p(x,N)$; $\tilde{K}G_n^p(x,M)$	$\tilde{K}G_n^p(x,M)=\sum_{m=0}^{n}\sum_{r,k=0}^{r+k\le n-m}\binom{n}{m}\frac{G_{n-m-r-k}\,(N-M)_k\,p^k\,(n-m)!}{(n-m-r-k)!\,(r+1)!\,k!}\,\tilde{K}B_m^p(x,N)$
VII.	$\tilde{K}H_m^{p,\nu}(x,N)$; $\tilde{K}E_n^p(x,M)$	$\tilde{K}E_n^p(x,M)=\sum_{m=0}^{n}\sum_{r,k=0}^{k+2r\le n-m}\binom{n}{m}\frac{E_{n-m-k-2r}\,(N-M)_k\,p^k\,\nu^r(n-m)!}{2^r(n-m-k-2r)!\,r!\,k!}\,\tilde{K}H_m^{p,\nu}(x,N)$
VIII.	$\tilde{K}H_m^{p,\nu}(x,N)$; $\tilde{K}G_n^p(x,M)$	$\tilde{K}G_n^p(x,M)=\sum_{m=0}^{n}\sum_{r=0}^{\left[\frac{n-m}{2}\right]}\binom{n}{m}\frac{G_{n-m-2r}\,\nu^r(n-m)!}{2^r(n-m-2r)!\,r!}\,\tilde{K}H_m^{p,\nu}(x,N)$

Similarly, in view of Theorem 2.2, the duplication formulas for certain members belonging to the Boas-Buck-Appell family mentioned in Table 3 are established. We list these formulas in Table

Table 5. Duplication formulas for certain members belonging to the Boas-Buck-Appell family

S.No.	${}_pA_n(x)$	Duplication formulas
I.	${}_{\mathfrak{B}}H_n^{\nu}((a)_p,(b)_q;x)$	${}_{\mathfrak{B}}H_n^{\nu}((a)_p,(b)_q;\alpha x)=\sum_{m=0}^{n}\sum_{k=0}^{\left[\frac{n-m}{2}\right]}\binom{n}{m}\frac{(1-\alpha)^{n-m-k}\,(-\nu)^k\alpha^m(n-m)!}{2^k k!\,(n-m-2k)!}\,{}_{\mathfrak{B}}H_m^{\nu}((a)_p,(b)_q;x)$
II.	${}_{\mathfrak{B}}B_n((a)_p,(b)_q;x)$	${}_{\mathfrak{B}}B_n((a)_p,(b)_q;\alpha x)=\sum_{m=0}^{n}\sum_{k,r=0}^{k+r\leq n-m}\binom{n}{m}\frac{B_{n-m-k-r}\,(1-\alpha)^r\,\alpha^{k+m}(n-m)!}{r!\,(k+1)!\,(n-m-k-r)!}\,{}_{\mathfrak{B}}B_m((a)_p,(b)_q;x)$
III.	${}_PH_n^{\lambda,\nu}(x)$	${}_PH_n^{\lambda,\nu}(\alpha x)=\sum_{m=0}^{n}\sum_{k,r=0}^{k+2r\leq n-m}\binom{n}{m}\frac{(\lambda)_{n-m-k-2r}\,(-\lambda)_k\,(\lambda)_m\,(\alpha-1)^r\alpha^{k+m}\nu^r(n-m)!}{2^r(\lambda)_n\;r!\;k!\,(n-m-k-2r)!}\,{}_PH_m^{\lambda,\nu}(x)$
IV.	${}_PB_n^{\lambda}(x)$	${}_PB_n^{\lambda}(\alpha x)=\sum_{m=0}^{n}\sum_{l,k,r=0}^{l+k+r\leq n-m}\binom{n}{m}\frac{B_{n-m-k-l-r}\,(-\lambda)_r\,(\lambda)_l\,(\lambda)_m\,\alpha^{k+m+r}(n-m)!}{(\lambda)_n\;r!\;l!\,(k+1)!\,(n-m-k-l-r)!}\,{}_PB_m^{\lambda}(x)$
V.	${}_{\tilde{K}}H_n^{p,\nu}(x,N)$	${}_{\tilde{K}}H_n^{p,\nu}(\alpha x,N)=\sum_{m=0}^{n}\sum_{k,r=0}^{k+2r\leq n-m}\binom{n}{m}\frac{(N)_{n-m-k-2r}\,(N)_k\,(\alpha-1)^r\alpha^{k+m}\nu^r p^{n-2r-m}(n-m)!}{2^r r!\;k!\,(n-m-k-2r)!}\,{}_{\tilde{K}}H_m^{p,\nu}(x,N)$
VI.	${}_{\tilde{K}}B_n^{p}(x,N)$	${}_{\tilde{K}}B_n^{p}(\alpha x,N)=\sum_{m=0}^{n}\sum_{l,k,r=0}^{l+k+r\leq n-m}\binom{n}{m}\frac{B_{n-m-k-l-r}\,(-N)_l\,(N)_r\,\alpha^{k+m+r}p^{l+r}(n-m)!}{r!\;l!\,(k+1)!\,(n-m-k-l-r)!}\,{}_{\tilde{K}}B_m^{p}(x,N)$

In view of Remarks 1.1 and 1.2, the connection and duplication formulas for several members related to the Brenke-Appell polynomials ${}_YA_n(x)$ and Boas-Buck polynomials $p_n(x)$ are obtained.

The literature on linearization and connection problems is extremely vast and numerous methods and approaches for computing the linearization and connection coefficients have been developed. In this article, we have developed a general method for obtaining the connection and duplication formulas associated with the Boas-Buck-Appell polynomials and these formulas can be used for solving differential equations with polynomial coefficients. In the forthcoming investigation, the linearization coefficients for the hybrid forms of the special polynomials are obtained.

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