Smarandache curves according to the Sabban frame belonging to spherical indicatrix curve of the Salkowski curve

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Abstract

In this study, Smarandache curves are defined according to the Sabban frame by means of the spherical indicatrix curve of the Salkowski curve. Then using these curves we calculated geodesic curvatures. Finally each curve is drawn with maple program.

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1 Introduction

Erich Salkowski (1881-1943), a German mathematician. In 1909, he defined curve families the non-constant torsion with constant curvature [8]. J. Monterde constructed a method for closed curves. The properties of Salkowski curves used first in [6]. Also for authors worked on the Salkowski curve, see in [1, 2, 5, 10, 11, 12]. When the Frenet vectors of any curve are taken as the position vector, the regular curves generated by these vectors are called Smarandache curves [15]. Smarandache curves in Euclidean 3-space are defined and some features of these curves are given in [3] and also for authors worked on the Smarandache curves, see in [9, 10]. In 1990, the geodesic curve of a spherical curve was calculated by J. Koenderink with the Sabban frame of the spherical indicator curves [7]. Then Smarandache curves obtained from Sabban frame are defined and geodesic curvatures of these curves are given in [14]. In this study, Smarandache curves are defined according to the Sabban frames belonging to the spherical indicator curves of each of the T, N, B Frenet vectors of the Salkowski curve. The geodesic curvatures of these curves are then calculated.

2 Preliminaries

In the Euclidean 3-space E^3, the Frenet frame of any curve α is given by $\{T, N, B\}$. For an arbitrary curve $\alpha \in E^3$, with first and second curvatures, κ and τ respectively, the Frenet apparatus are given by [4]

$$T' = \kappa N, \quad N' = -\kappa T + \tau B, \quad B' = -\tau N.$$

Accordingly, the spherical indicatrix curves of Frenet vectors are (T), (N) and (B) respectively. These equations of curves are given by [4]

$$\alpha_T(s) = T(s), \quad \alpha_N(s) = N(s), \quad \alpha_B(s) = B(s). \tag{2.1}$$

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Let $\gamma : I \to S^2$ be a unit speed spherical curve. We denote s as the arc-length parameter of γ. Let us denote by

$$\gamma(s) \quad = \quad \gamma(s), \quad t(s) = \gamma'(s), \quad d(s) = \gamma(s) \wedge t(s). \tag{2.2}$$

We call $t(s)$ a unit tangent vector of γ. $\{\gamma, t, d\}$ frame is called the Sabban frame of γ on S^2 [4]. Then we have the following spherical Frenet formulae of γ :

$$\gamma' \quad = \quad t, \quad t' = -\gamma + \kappa_g d, \quad d' = -\kappa_g t \tag{2.3}$$

where is called the geodesic curvature of κ_g on S^2 and

$$\kappa_g \quad = \quad \langle t', d \rangle \ [7] \tag{2.4}$$

Definition 2.1. (Salkowski curve) [8]. For any $m \in \mathbb{R}$ with $m \neq \mp\frac{1}{\sqrt{3}}$, 0, let us define the space curve

$$\begin{aligned} \gamma_m(s) \quad = \quad & \frac{1}{\sqrt{1+m^2}}\Bigg(-\frac{1-n}{4(1+2n)}\sin((1+2n)s) - \frac{1+n}{4(1-2n)}\sin((1-2n)s) - \frac{1}{2}\sin(s), \\ & \frac{1-n}{4(1+2n)}\cos((1+2n)s) + \frac{1+n}{4(1-2n)}\cos((1-2n)s) + \frac{1}{2}\cos(s), \frac{1}{4m}\cos(2ns)\Bigg) \end{aligned}$$

where $n = \frac{m}{\sqrt{1+m^2}}$. The Frenet apparatus are

$$\begin{cases} \kappa \quad = \quad 1, \quad \tau = \tan(ns), \quad \|\gamma_m(s)\| = \frac{cos(ns)}{\sqrt{1+m^2}}, \\ T(s) \quad = \quad \Big(-\cos(s)\cos(ns) - n\sin(s)\sin(ns), -\sin(s)\cos(ns) + n\cos(s)\sin(ns), \\ \qquad -\frac{n}{m}\sin(ns)\Big), \\ N(s) \quad = \quad n\Big(\frac{\sin(s)}{m}, -\frac{\cos(s)}{m}, -1\Big), \\ B(s) \quad = \quad \Big(-\cos(s)\sin(ns) + n\sin(s)\cos(ns), -\sin(s)\sin(ns) - n\cos(s)\cos(ns), \\ \qquad \frac{n}{m}\cos(ns)\Big). \end{cases} \tag{2.5}$$

The shapes of these curves are given in Figure 1.

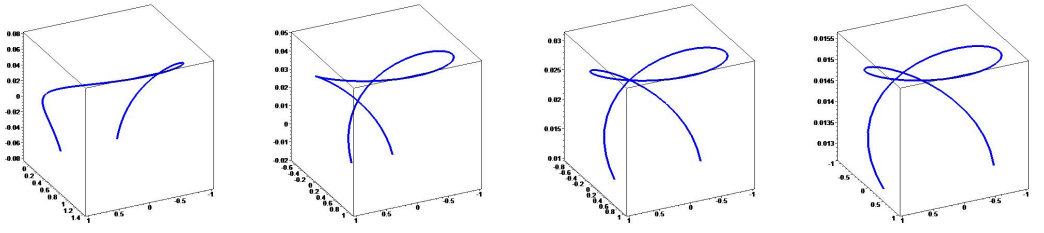


Figure 1: Salkowski curve, $m = \frac{1}{3}, \frac{1}{5}, \frac{1}{8}, \frac{1}{16}$ and $s = [-5, 5]$

Let $(\alpha), (\delta)$ and (ζ) be spherical indicatrix curves of tangent, principal normal and binormal vectors belonging to Salkowski curve, respectively. Using the equations (2.3) and (2.4), Sabban apparatus belonging to these curves is given by

$$\begin{cases} T & = & T\,, \quad T_T = N\,, \quad T \wedge T_T = B, \\ T' & = & T_T, \quad T'_T = -T + \tan(ns)(T \wedge T_T), \\ (T \wedge T_T)' & = & -\tan(ns)T_T, \quad K_g^T = \tan(ns). \end{cases} \tag{2.6}$$

$$\begin{cases} N & = & N, \quad T_N = -\cos(ns)T + \sin(ns)B\,, \quad , \\ N \wedge T_N & = & \sin(ns)T + \cos(ns)B, \quad N' = T_N, \\ T'_N & = & -N + n\sec(ns)(N \wedge T_N), \quad (N \wedge T_N)' = n\sec(ns)T_N, \\ K_g^N & = & n\sec(ns). \end{cases} \tag{2.7}$$

$$\begin{cases} N(s) & = & \left(\frac{n\sin(s)}{m}, -\frac{n\cos(s)}{m}, -n\right), \\ T_N(s) & = & -\cos(ns)T + \sin(ns)B, \\ (N \wedge T_N)(s) & = & \sin(ns)T + \cos(ns)B. \end{cases}$$

$$\begin{cases} B = B, & T_B = -N, \quad B \wedge T_B = T, \\ B' = T_B\,, & T'_B = -B + \cot(ns)(B \wedge T_B)\,, \\ (B \wedge T_B)' & = & \cot(ns)T_B, \quad K_g^B = \cot(ns). \end{cases} \tag{2.8}$$

$$\begin{cases} B(s) & = \Big(-\cos(s)\sin(ns) + n\sin(s)\cos(ns), -\sin(s)\sin(ns) \\ & \quad -n\cos(s)\cos(ns), \frac{n}{m}\cos(ns)\Big), \\ T_B(s) & = n\left(-\frac{\sin(s)}{m}, +\frac{\cos(s)}{m}, +1\right), \\ (B\wedge T_B)(s) & = \Big(-\cos(s)\cos(ns) - n\sin(s)\sin(ns), -\sin(s)\cos(ns) \\ & \quad +n\cos(s)\sin(ns), -\frac{n}{m}\sin(ns)\Big). \end{cases}$$

3 Smarandache curves according to the Sabban frame belonging to spherical indicatrix curve of the Salkowski curve

Definition 3.1. Let $\alpha = \alpha(s)$ be a curve in S^2 and $\{T, T_T, T\wedge T_T\}$ be the Sabban frame of this curve. Then TT_T-Smarandache curve is given by

$$\alpha_1(s) = \frac{1}{\sqrt{2}}(T + T_T). \tag{3.1}$$

According to equation (2.6) we can parameterize the $\alpha_1(s)$-Smarandache curve as in the following form

$$\begin{aligned} \alpha_1(s) &= \frac{1}{\sqrt{2}}\Big(-\cos(s)\cos(ns) - n\sin(s)\sin(ns) + \frac{n}{m}\sin(s), \\ &\quad -\sin(s)\cos(ns) + n\cos(s)\sin(ns) - \frac{n}{m}\cos(s), -\frac{n}{m}\sin(ns) - n\Big). \end{aligned}$$

The shapes for these curves are given in Figure 2.

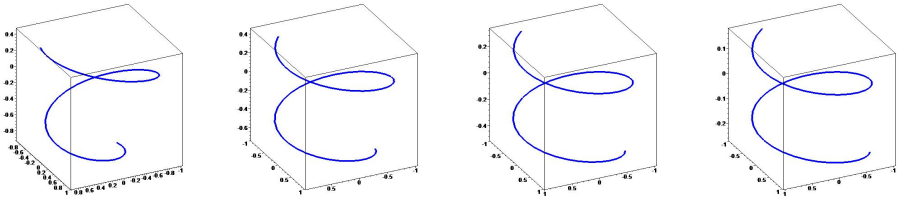


Figure 2: α_1-Smarandache curve, $m = \frac{1}{3}, \frac{1}{5}, \frac{1}{8}, \frac{1}{16}$ and $s = [-5, 5]$

Theorem 3.2. The geodesic curvature $K_g^{\alpha_1}$ according to $\alpha_1(s)$-Smarandache curve is

$$K_g^{\alpha_1} = \frac{\sqrt{2}\,(-\tan(ns)\sigma_2 + \sigma_3)}{(2+\tan^2(ns))^{\frac{5}{2}}},$$

where the coefficients σ_2 and σ_3 are

$$\begin{aligned} \sigma_2 &= -2 - n\tan(ns)\sec(ns) - 3\tan^2(ns) - \tan^4(ns), \\ \sigma_3 &= 2\tan(ns) + 2n\sec(ns) + \tan^3(ns). \end{aligned}$$

Proof. If we take the derivative in equation (3.1) we get and from the equation (2.6)

$$T_{\alpha_1}\frac{ds^*}{ds} = \frac{1}{\sqrt{2}}(-T + T_T + \tan(ns)(T \wedge T_T)), \tag{3.2}$$

if we take the norm of (3.2),

$$\frac{ds^*}{ds} = \frac{1}{\sqrt{2}}\sqrt{2 + \tan^2(ns)}.$$

We obtained the tangent of $\alpha_1(s)$-Smarandahce curve as in

$$T_{\alpha_1} = \frac{1}{\sqrt{2 + \tan^2(ns)}}(-T + T_T + \tan(ns)(T \wedge T_T)). \tag{3.3}$$

The derivative of (3.3) is

$$T'_{\alpha_1} = \frac{\sqrt{2}}{(2 + \tan^2(ns))^2}(\sigma_1 T + \sigma_2 T_T + \sigma_3(T \wedge T_T)). \tag{3.4}$$

and

$$(T \wedge T_{\alpha_1}) = \frac{1}{\sqrt{2 + \tan^2(ns)}}(-\tan(ns)T_T + T \wedge T_T) \tag{3.5}$$

where the coefficients σ_1, σ_2 and σ_3 are

$$\begin{aligned}
\sigma_1 &= -2 - \tan^2(ns) + \tan(ns)\tan(ns)', \\
\sigma_2 &= -2 - \tan(ns)\tan(ns)' - 3\tan^2(ns) - \tan^4(ns), \\
\sigma_3 &= 2\tan(ns) + 2\tan(ns)' + \tan^3(ns).
\end{aligned}$$

From the equation (2.4), (3.4) and (3.5), the geodesic curvature of α_1 is obtained.

Q.E.D.

Definition 3.3. Let $\alpha = \alpha(s)$ be a curve and $\{T, T_T, T \wedge T_T\}$ be Sabban frame of this curve. Then $T(T \wedge T_T)$-Smarandache curve is given by

$$\alpha_2(s) = \frac{1}{\sqrt{2}}(T + (T \wedge T_T)). \tag{3.6}$$

According to equation (2.6) we can parameterize the $\alpha_2(s)$-Smarandache curve as in the following way

$$\begin{aligned}
\alpha_2(s) = \frac{1}{\sqrt{2}}\Big(& -\cos(s)(\cos(ns) + \sin(ns)) + n\sin(s)(\cos(ns) - \sin(ns)), \\
& -\sin(s)(\cos(ns) + \sin(ns)) - n\cos(s)(\cos(ns) - \sin(ns)), \\
& \frac{n}{m}(\cos(ns) + \sin(ns))\Big).
\end{aligned}$$

The shapes for this curves are given in Figure 3.

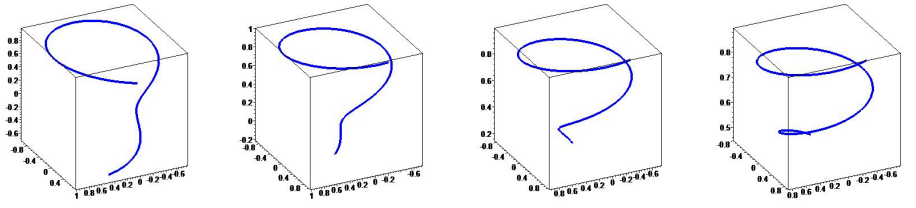


Figure 3: α_2-Smarandache curve, $m = \frac{1}{3}, \frac{1}{5}, \frac{1}{8}, \frac{1}{16}$ and $s = [-5, 5]$

Theorem 3.4. The geodesic curvature $K_g^{\alpha_2}$ according to $\alpha_2(s)$-Smarandache curve is given by

$$K_g^{\alpha_2}(s) = \frac{\sqrt{2}\tan(ns)}{1-\tan(ns)}.$$

Proof. If we take the derivative of (3.6) and from the equation (2.6) we have

$$T_{\alpha_2}\frac{ds^*}{ds} = \frac{1}{\sqrt{2}}(T_T - \tan(ns)T_T). \tag{3.7}$$

We take the norm of (3.7),

$$\frac{ds^*}{ds} = \frac{1-\tan(ns)}{\sqrt{2}}.$$

We obtained the tangent of $\alpha_2(s)$-Smarandahce curve as in

$$T_{\alpha_2} = T_T. \tag{3.8}$$

The derivative of (3.8) is equal to

$$T'_{\alpha_2} = \frac{\sqrt{2}(-T + \tan(ns)(T \wedge T_T))}{1-\tan(ns)}. \tag{3.9}$$

and

$$(T \wedge T_{\alpha_2}) = T \wedge T_T. \tag{3.10}$$

From the equation (2.4), (3.9) and (3.10), the geodesic curvature of α_2 is obtained. Q.E.D.

Definition 3.5. Let $\alpha = \alpha(s)$ be a curve and $\{T, T_T, T \wedge T_T\}$ be Sabban frame of this curve. Then $T_T(T \wedge T_T)$-Smarandache curve is given by

$$\alpha_3(s) = \frac{1}{\sqrt{2}}(T_T + (T \wedge T_T)). \tag{3.11}$$

According to equation (2.6) we can parameterize the $\alpha_3(s)$-Smarandache curve as in the following form

$$\begin{aligned}\alpha_3(s) \quad = \quad & \frac{1}{\sqrt{2}}\Big(-\cos(s)\sin(ns) + n\sin(s)\cos(ns) + \frac{n}{m}\sin(s), \\ & -\sin(s)\sin(ns) - n\cos(s)\cos(ns) - \frac{n}{m}\cos(s), \frac{n}{m}\cos(ns) - n\Big).\end{aligned}$$

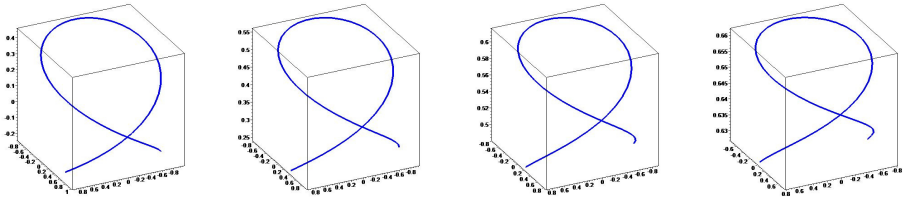


Figure 4: α_3-Smarandache curve, $m = \frac{1}{3}, \frac{1}{5}, \frac{1}{8}, \frac{1}{16}$ and $s = [-5, 5]$

The shapes for this curves are given in Figure 4.

Theorem 3.6. The geodesic curvature $K_g^{\alpha_3}$ according to $\alpha_3(s)$-Smarandache curve is given by

$$K_g^{\alpha_3}(s) = \frac{\tan(ns)\sigma_4 - \sigma_5 + \sigma_6}{(1 + 2\tan^2(ns))^{\frac{5}{2}}}$$

where the coefficients σ_4, σ_5 and σ_6 are

$$\begin{aligned} \sigma_4 &= 2n\tan(ns)\sec(ns) + \tan(ns) + 2\tan^3(ns), \\ \sigma_5 &= -1 - n\sec(ns) - 3\tan^2(ns) - 2\tan^4(ns), \\ \sigma_6 &= -\tan^2(ns) + n\sec(ns) - 2\tan^4(ns). \end{aligned}$$

Proof. If we take the derivative of (3.11) and from the equation (2.6) we get

$$T_{\alpha_3}\frac{ds^*}{ds} = \frac{1}{\sqrt{2}}(-T - \tan(ns)T_T + \tan(ns)(T \wedge T_T)), \tag{3.12}$$

We take the norm of (3.12),

$$\frac{ds^*}{ds} = \frac{1}{\sqrt{2}}\sqrt{1 + 2\tan^2(ns)}.$$

We obtain the tangent of $\alpha_3(s)$-Smarandahce curve as in

$$T_{\alpha_3} = \frac{1}{\sqrt{1 + 2\tan^2(ns)}}(-T - \tan(ns)T_T + \tan(ns)(T \wedge T_T)). \tag{3.13}$$

The derivative of (3.13) is given by

$$T'_{\alpha_3} = \frac{\sqrt{2}}{(1 + 2\tan^2(ns))^2}(\sigma_4 T + \sigma_5 T_T + \sigma_6(T \wedge T_T)) \tag{3.14}$$

and

$$T \wedge T_{\alpha_3} = \frac{-\tan(ns)}{\sqrt{1 + 2\tan^2(ns)}}(T_T + T \wedge T_T). \tag{3.15}$$

From the equation (2.4), (3.14) and (3.15), the geodesic curvature of α_3 is obtained. Q.E.D.

Definition 3.7. Let $\alpha = \alpha(s)$ be a curve and $\{T, T_T, T \wedge T_T\}$ be Sabban frame of this curve. Then $TT_T(T \wedge T_T)$-Smarandache curve is given by

$$\alpha_4(s) = \frac{1}{\sqrt{3}}(T + T_T + (T \wedge T_T)). \tag{3.16}$$

According to equation (2.7) we can parameterize the $\alpha_4(s)$-Smarandache curve as in the following form

$$\begin{aligned}\alpha_4(s) \quad = \quad & \frac{1}{\sqrt{3}}\Big(-\cos(s)(\cos(ns)+\sin(ns)) + n\sin(s)(\cos(ns)-\sin(ns)) \\ & +\frac{n}{m}\sin(s), -\sin(s)(\cos(ns)+\sin(ns)) - \frac{n}{m}\cos(s) \\ & -n\cos(s)(\cos(ns)-\sin(ns)), \frac{n}{m}(\cos(ns)-\sin(ns)) - n\Big).\end{aligned}$$

The shapes for these curves are given in Figure 5.

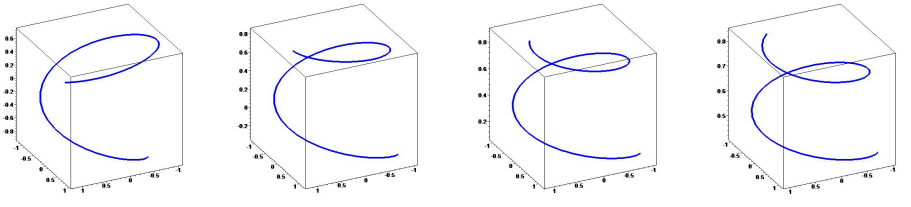


Figure 5: α_4-Smarandache curve, $m = \frac{1}{3}, \frac{1}{5}, \frac{1}{8}, \frac{1}{16}$ and $s = [-5, 5]$

Theorem 3.8. The geodesic curvature $K_g^{\alpha_4}$ according to $\alpha_4(s)$-Smarandache curve is given as

$$K_g^{\alpha_4} = \frac{\sqrt{3}\,(-\tan(ns)\sigma_8 + (1-\tan(ns))\sigma_9)}{((2-2\tan(ns)) + 2\tan(ns)^2)^{\frac{5}{2}}},$$

where the coefficients σ_8 and σ_9 are

$$\begin{aligned}\sigma_8 \quad = \quad & -n\sec(ns) - n\tan(ns)\sec(ns) - 2 - 4\tan^2(ns) + 2\tan(ns) \\ & +2\tan^3(ns) - 2\tan^4(ns),\end{aligned}$$

$$\begin{aligned}\sigma_9 \quad = \quad & -n\tan(ns)\sec(ns) + 2\tan(ns) - 4\tan^2(ns) + 2n\sec(ns) + 4\tan^3(ns) \\ & -2\tan^4(ns).\end{aligned}$$

Proof. If we take the derivative of (3.16) and from the equation (2.6) we get

$$T_{\alpha_4}\frac{ds^*}{ds} = \frac{1}{\sqrt{3}}(-T + (1-\tan(ns))T_T + \tan(ns)(T \wedge T_T)). \tag{3.17}$$

We take the norm of (3.17), we get

$$\frac{ds^*}{ds} = \frac{1}{\sqrt{3}}\sqrt{2(1-\tan(ns)+\tan^2(ns))}.$$

We obtain the tangent of $\alpha_4(s)$-Smarandahce curve as in

$$T_{\alpha_4} = \frac{-T+(1-\tan(ns))T_T+\tan(ns)(T\wedge T_T)}{\sqrt{2(1-\tan(ns)+\tan^2(ns))}}. \tag{3.18}$$

The derivative of (3.18) is

$$T'_{\alpha_4} = \frac{\sqrt{3}}{4(1-\tan(ns)+\tan^2(ns))^2}(\sigma_7 T+\sigma_8 T_T+\sigma_9(T\wedge T_T)) \tag{3.19}$$

and

$$T\wedge T_{\alpha_4} = \frac{-\tan(ns)T_T+(1-\tan(ns))(T\wedge T_T)}{\sqrt{2(1-\tan(ns)+\tan^2(ns))}} \tag{3.20}$$

where the coefficients σ_7, σ_8 and σ_9 are

$$\begin{aligned}
\sigma_7 &= -n\sec(ns)+2n\tan(ns)\sec(ns)-2+2\tan(ns)-2\tan^2(ns)\\
&\quad +2\tan^3(ns),\\
\sigma_8 &= -n\sec(ns)-n\tan(ns)\sec(ns)-2-4\tan^2(ns)+2\tan(ns)\\
&\quad +2\tan^3(ns)-2\tan^4(ns),\\
\sigma_9 &= -n\tan(ns)\sec(ns)+2\tan(ns)-4\tan^2(ns)+2n\sec(ns)+4\tan^3(ns)\\
&\quad -2\tan^4(ns).
\end{aligned}$$

From the equation (2.4), (3.19) and (3.20), the geodesic curvature of α_4 is obtained. Q.E.D.

Definition 3.9. Let $\delta = \delta(s)$ be a curve and $\{N, T_N, N\wedge T_N\}$ be Sabban frame of this curve. Then NT_N-Smarandache curve is given by

$$\delta_1(s) = \frac{1}{\sqrt{2}}(N+T_N). \tag{3.21}$$

According to equation (2.7) we can parameterize the $\delta_1(s)$-Smarandache curve as in the following form

$$\delta_1(s) = \frac{1}{\sqrt{2}}(-\cos(ns)T+N+\sin(ns)B).$$

The shapes for these curves are given in Figure 6.

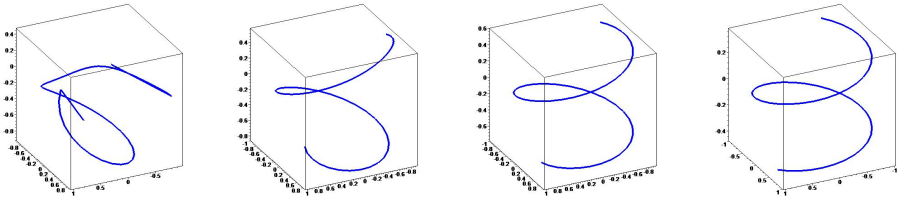


Figure 6: δ_1-Smarandache curves, $m = \frac{1}{3}, \frac{1}{5}, \frac{1}{8}, \frac{1}{16}$ and $s = [-5, 5]$

Theorem 3.10. The geodesic curvature $K_g^{\delta_1}$ according to $\delta_1(s)$-Smarandache curve is given by

$$K_g^{\delta_1} = \frac{\sqrt{2}\,(-n\cos(ns)\sigma_{11} + \sigma_{12})}{(2 + n^2\cos^2(ns))^{\frac{5}{2}}},$$

where the coefficients σ_{11} and σ_{12} are

$$\begin{aligned} \sigma_{11} &= \frac{n^3}{2}\sin(2ns) - 3n^2\cos^2(ns) + n^4\cos^4(ns) + 2 \\ \sigma_{12} &= n^4\cos^2(ns)\sin(2ns) + (2n + n^3\cos^2(ns))(\cos(ns) - \sin(ns)). \end{aligned}$$

Proof. If we take the derivative of (3.22) and from the equation (2.7) we get

$$T_{\delta_1}\frac{ds^*}{ds} = \frac{1}{\sqrt{2}}(-N + T_N + n\cos(ns)(N \wedge T_N)). \tag{3.22}$$

If take the norm of (3.22), we have

$$\frac{ds^*}{ds} = \frac{\sqrt{2 + n^2\cos^2(ns))}}{\sqrt{2}}.$$

We obtain the tangent of $\delta_1(s)$-Smarandahce curve as in

$$T_{\delta_1} = \frac{-N + T_N + n\cos(ns)(N \wedge T_N)}{\sqrt{2 + n^2\cos^2(ns)}}. \tag{3.23}$$

The derivative of (3.22) is given as

$$T'_{\delta_1} = \frac{\sqrt{2}(\sigma_{10}N + \sigma_{11}T_N + \sigma_{12}(N \wedge T_N))}{(2 + n^2\cos^2(ns))^2} \tag{3.24}$$

and

$$N \wedge T_{\delta_1} = \frac{-n\cos(ns)T_N + (N \wedge T_N)}{\sqrt{2 + n^2\cos^2(ns)}} \tag{3.25}$$

where the coefficients σ_{10}, σ_{11} and σ_{12} are

$$\begin{aligned}
\sigma_{10} &= \frac{n^3}{2}\sin(2ns) - n^2\cos^2(ns) - 2,\\
\sigma_{11} &= \frac{n^3}{2}\sin(2ns) - 3n^2\cos^2(ns) + n^4\cos^4(ns) + 2\\
\sigma_{12} &= n^4\cos^2(ns)\sin(2ns) + (2n + n^3\cos^2(ns))(\cos(ns) - \sin(ns)).
\end{aligned}$$

From the equation (2.4), (3.24) and (3.25), the geodesic curvature of δ_1 is obtained. Q.E.D.

Definition 3.11. Let $\delta = \delta(s)$ be a curve and $\{N, T_N, N \wedge T_N\}$ be Sabban frame of this curve.

$$\delta_2(s) = \frac{1}{\sqrt{2}}(N + (N \wedge T_N)). \tag{3.26}$$

According to equation (2.7) we can parameterize the $\delta_2(s)$-Smarandache curve as in the following form

$$\delta_2(s) = \frac{1}{\sqrt{2}}(\sin(ns)T + N + \cos(ns)B)$$

The shape of this curve is given in Figure 7.

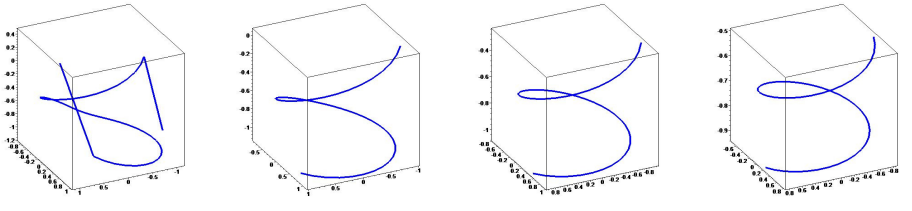


Figure 7: δ_2-Smarandache curves, $m = \frac{1}{3}, \frac{1}{5}, \frac{1}{8}, \frac{1}{16}$ and $s = [-5, 5]$

Theorem 3.12. The geodesic curvature $K_g^{\delta_2}$ according to $\delta_2(s)$-Smarandache curve is given by

$$K_g^{\delta_2} = \frac{\sqrt{2}n\cos(ns)}{1 - n\cos(ns)}.$$

Proof. If we take the derivative of (3.26) and from the equation (2.7) we get

$$T_{\delta_2}\frac{ds^*}{ds} = \frac{1}{\sqrt{2}}(1 - n\cos(ns))T_N. \tag{3.27}$$

If take the norm of (3.27), we get

$$\frac{ds^*}{ds} = \frac{1 - n\cos(ns)}{\sqrt{2}}.$$

We obtain the tangent vector of $\delta_2(s)$-Smarandahce curve as in

$$T_{\delta_2} = T_N. \tag{3.28}$$

The derivative of (3.28) is

$$T'_{\delta_2} = \frac{\sqrt{2}(-N + n\cos(ns)(N \wedge T_N))}{1 - n\cos(ns)}. \tag{3.29}$$

and

$$N \wedge T_{\delta_2} = N \wedge T_N. \tag{3.30}$$

From the equation (2.4), (3.29) and (3.30), the geodesic curvature of δ_2 is obtained. Q.E.D.

Definition 3.13. Let $\delta = \delta(s)$ be a curve and $\{N, T_N, N \wedge T_N\}$ be Sabban frame of this curve. Then $T_N(N \wedge T_N)$-Smarandache curve ($\delta_3(s)$-Smarandache curve) is given by

$$\delta_3(s) = \frac{1}{\sqrt{2}}(T_N + (N \wedge T_N)). \tag{3.31}$$

According to equation (2.7) we can parameterize the $\delta_3(s)$-Smarandache curve as in the following form

$$\delta_3(s) = (\sin(ns) - \cos(ns))T + (\sin(ns) + \cos(ns))B.$$

The shape for this curve is given in Figure 8.

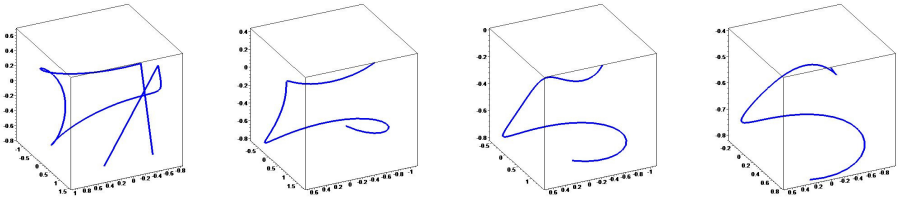


Figure 8: δ_3-Smarandache curves, $m = \frac{1}{3}, \frac{1}{5}, \frac{1}{8}, \frac{1}{16}$ and $s = [-5, 5]$

Theorem 3.14. The geodesic curvature $K_g^{\delta_3}$ according to $\delta_3(s)$-Smarandache curve is given by

$$K_g^{\delta_3} = \frac{-\sqrt{2}n\cos(ns)\,(\sigma_{14}+\sigma_{15})}{\left((1+2n^2\cos^2(ns)\right)^{\frac{5}{2}}},$$

where the coefficients σ_{14} and σ_{15} are

$$\begin{aligned}\sigma_{14} &= -1+n^2\sin(ns)-3n^2\cos^2(ns)-2n^4\cos^4(ns)\\ \sigma_{15} &= -n^2\sin(ns)-n^2\cos^2(ns)-2n^4\cos^4(ns).\end{aligned}$$

Proof. If we take the derivative of (3.31) and from the equation (2.7) we get

$$T_{\delta_3}\frac{ds^*}{ds} = \frac{1}{\sqrt{2}}(-N-n\cos(ns)T_N+n\cos(ns)(N\wedge T_N)). \tag{3.32}$$

If take the norm of (3.32) we have

$$\frac{ds^*}{ds} = \frac{\sqrt{1+2n^2\cos^2(ns)}}{\sqrt{2}}.$$

We obtain the tangent vector of $\delta_3(s)$-Smarandahce curve as in

$$T_{\delta_3} = \frac{-N-n\cos(ns)T_N+n\cos(ns)(N\wedge T_N))}{\sqrt{1+2n^2\cos^2(ns)}}. \tag{3.33}$$

The derivative of (3.33) is given as

$$T'_{\delta_3} = \frac{\sqrt{2}(\sigma_{13}N+\sigma_{14}T_N+\sigma_{15}(N\wedge T_N)}{(1+2n^2\cos^2(ns))^2}. \tag{3.34}$$

and

$$N\wedge T_{\delta_3} = \frac{-n\cos(ns)(T_N+N\wedge T_N)}{\sqrt{1+2n^2\cos^2(ns)}} \tag{3.35}$$

where the coefficients σ_{13}, σ_{14} and σ_{15} are

$$\begin{aligned}\sigma_{13} &= -n^3\sin(2ns)+n\cos(ns)+2n^3\cos^3(ns),\\ \sigma_{14} &= -1+n^2\sin(ns)-3n^2\cos^2(ns)-2n^4\cos^4(ns)\\ \sigma_{15} &= -n^2\sin(ns)-n^2\cos^2(ns)-2n^4\cos^4(ns).\end{aligned}$$

From the equation (2.4), (3.34) and (3.35), the geodesic curvature of δ_3 is obtained. Q.E.D.

Definition 3.15. Let $\delta=\delta(s)$ be a curve and $\{N, T_N, N\wedge T_N\}$ be Sabban frame of this curve. Then $NT_N(N\wedge T_N)$-Smarandache curve ($\delta_4(s)$-Smarandache curve) is given by

$$\delta_4(s) = \frac{1}{\sqrt{3}}(N+T_N+(N\wedge T_N)). \tag{3.36}$$

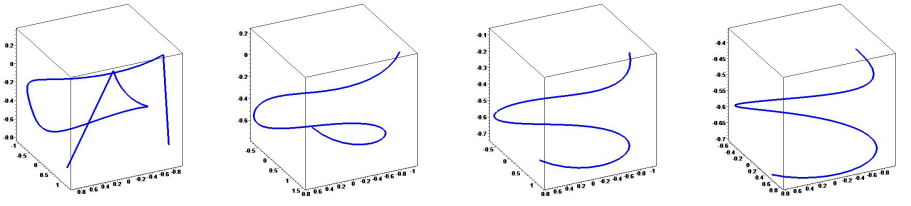


Figure 9: δ_4-Smarandache curves, $m = \frac{1}{3}, \frac{1}{5}, \frac{1}{8}, \frac{1}{16}$ and $s = [-5, 5]$

According to equation (2.7) we can parameterize the $\delta_4(s)$-Smarandache curve as in the following form

$$\delta_4(s) = \frac{1}{\sqrt{3}}\Big((\sin(ns) - \cos(ns))T + N + (\sin(ns) + \cos(ns))B\Big).$$

The shape of this curve is given in Figure 9.

Theorem 3.16. The geodesic curvature $K_g^{\delta_4}$ according to $\delta_4(s)$-Smarandache curve is given by

$$K_g^{\delta_4} = \frac{\sqrt{2}\,(-n\cos(ns)\sigma_{17} + (1 - n\cos(ns))\sigma_{18})}{(2 - 2n\cos(ns) + 2n^2\cos^2(ns))^{\frac{5}{2}}},$$

where the coefficients σ_{17} and σ_{18} are

$$\begin{aligned}
\sigma_{17} &= n\sin(ns)(-1 + 2n\cos(ns))(n\cos(ns) - 1) - (1 - n\cos(ns) + n^2\cos^2(ns)) \\
&\quad .(2 - 2n\cos(ns) + 2n^2\cos^2(ns))) \\
\sigma_{18} &= -n^2\sin(ns)(2n\cos^2(ns) - \sin(ns)) + (2n - 2n^2\cos(ns) + 2n^3\cos^2(ns)) \\
&\quad .(\cos(ns) - \sin(ns) - 2n\cos^2(ns)).
\end{aligned}$$

Proof. If we take the derivative of (3.36) and from the equation (2.7) we have

$$T_{\delta_4}\frac{ds^*}{ds} = \frac{1}{\sqrt{3}}(-N + (1 - n\cos(ns))T_N + n\cos(ns)(N \wedge T_N)) \tag{3.37}$$

If take the norm of (3.37), we get

$$\frac{ds^*}{ds} = \frac{\sqrt{2 - 2n\cos(ns) + 2n^2\cos^2(ns)}}{\sqrt{3}}.$$

We obtain the tangent vector of $\delta_4(s)$-Smarandahce curve as in

$$T_{\delta_4} = \frac{-N + (1 - n\cos(ns))T_N + n\cos(ns)(N \wedge T_N)}{\sqrt{2 - 2n\cos(ns) + 2n^2\cos^2(ns)}}. \tag{3.38}$$

The derivative of (3.38) is given by

$$T'_{\delta_4} = \frac{\sqrt{3}(\sigma_{16}N + \sigma_{17}T_N + \sigma_{18}N \wedge T_N)}{(2 - 2n\cos(ns) + 2n^2\cos^2(ns))^2} \tag{3.39}$$

and

$$N \wedge T_{\delta_4} = \frac{-n\cos(ns)T_N + (1 - n\cos(ns))N \wedge T_N}{\sqrt{2 - 2n\cos(ns) + 2n^2\cos^2(ns)}} \tag{3.40}$$

where the coefficients σ_{16}, σ_{17} and σ_{18} are

$$\begin{aligned}
\sigma_{16} &= -n\sin(ns)(1 - 2n\cos(ns)) + (n\cos(ns) - 1)(2 - 2n\cos(ns) + 2n^2\cos^2(ns))) \\
\sigma_{17} &= n\sin(ns)(-1 + 2n\cos(ns))(n\cos(ns) - 1) - (1 - n\cos(ns) + n^2\cos^2(ns)) \\
&\quad .(2 - 2n\cos(ns) + 2n^2\cos^2(ns))) \\
\sigma_{18} &= -n^2\sin(ns)(2n\cos^2(ns) - \sin(ns)) + (2n - 2n^2\cos(ns) + 2n^3\cos^2(ns)) \\
&\quad .(\cos(ns) - \sin(ns) - 2n\cos^2(ns)).
\end{aligned}$$

From the equation (2.4), (3.39) and (3.40), the geodesic curvature of δ_4 is obtained. Q.E.D.

Definition 3.17. Let $\zeta = \zeta(s)$ be a curve and $\{B, T_B, B \wedge T_B\}$ be Sabban frame of this curve. Then BT_B-Smarandache curve ($\zeta_1(s)$-Smarandache curve) is given by

$$\zeta_1(s) = \frac{1}{\sqrt{2}}(B + T_B). \tag{3.41}$$

According to equation (2.8) we can parameterize the $\zeta_1(s)$-Smarandache curve as in the following form

$$\begin{aligned}
\zeta_1(s) &= \frac{1}{\sqrt{2}}\Big(-\cos(s)\sin(ns) + n\sin(s)\cos(ns) + \frac{n}{m}\sin(s), \\
&\quad -\sin(s)\sin(ns) - n\cos(s)\cos(ns) - \frac{n}{m}\cos(s), \\
&\quad \frac{n}{m}\cos(ns) - n\Big).
\end{aligned}$$

The shape for this curve is given in Figure 10.

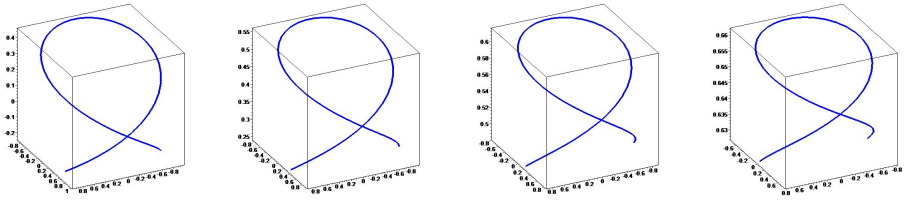


Figure 10: ζ_1-Smarandache curve, $m = \frac{1}{3}, \frac{1}{5}, \frac{1}{8}, \frac{1}{16}$ and $s = [-5, 5]$

Theorem 3.18. The geodesic curvature $K_g^{\zeta_1}$ according to $\zeta_1(s)$-Smarandache curve is

$$K_g^{\zeta_1} = \frac{\tan^4(ns)}{(2\tan^2(ns) + 1)^{\frac{5}{2}}}\left(\sigma_{19} - \sigma_{20} + 2\tan(ns)\sigma_{21}\right),$$

where the coefficients σ_{19}, σ_{20} and σ_{21} are

$$\begin{aligned}\sigma_{19} &= -n\cot(ns)\csc(ns) - 2 - \cot^2(ns),\\ \sigma_{20} &= -n\cot(ns)\csc(ns) + 2 + 3\cot^2(ns) + \cot^4(ns),\\ \sigma_{21} &= -2n\csc(ns) + 2\cot(ns) + \cot^3(ns).\end{aligned}$$

Proof. If we take the derivative of (3.41) and from the equation (2.8) we get

$$T_{\zeta_1}\frac{ds^*}{ds} = \frac{1}{\sqrt{2}}(-B + T_B + \cot(ns)B \wedge T_B). \tag{3.42}$$

If we take the norm of (3.42) we get

$$\frac{ds^*}{ds} = \frac{\sqrt{2 + \cot^2(ns)}}{\sqrt{2}}.$$

We obtained the tangent of $\zeta_1(s)$-Smarandahce curve as in

$$T_{\zeta_1} = \frac{-B + T_B + \cot(ns)B \wedge T_B}{\sqrt{2 + \cot^2(ns)}}. \tag{3.43}$$

The derivative of (3.43) is given by

$$T'_{\zeta_1} = \frac{\sqrt{2}(\sigma_{19}B - \sigma_{20}T_B + \sigma_{21}(B \wedge T_B))}{(2 + \cot^2(ns))^2} \tag{3.44}$$

and

$$B \wedge T_{\zeta_1} = \frac{-\cot(ns)T_B + B \wedge T_B}{\sqrt{2 + \cot^2(ns)}}. \tag{3.45}$$

From the equation (2.4), (3.44) and (3.45), the geodesic curvature of ζ_1 is obtained. Q.E.D.

Definition 3.19. Let $\zeta = \zeta(s)$ be a curve and $\{B, T_B, B \wedge T_B\}$ be Sabban frame of this curve. Then $B(B \wedge T_B)$-Smarandache curve ($\zeta_2(s)$-Smarandache curve) is given by

$$\zeta_2(s) = \frac{1}{\sqrt{2}}(B + (B \wedge T_B)). \tag{3.46}$$

According to equation (2.8) we can parameterize the $\zeta_2(s)$-Smarandache curve as in the following form

$$\begin{aligned}\zeta_2(s) = \frac{1}{\sqrt{2}}\Big(&\cos(s)(\cos(ns) - \sin(ns)) + n\sin(s)(\cos(ns) + \sin(ns)),\\ &\sin(s)(\cos(ns) - \sin(ns)) - n\cos(s)(\cos(ns) + \sin(ns)),\\ &\frac{n}{m}(\cos(ns) + \sin(ns))\Big).\end{aligned}$$

The shape for this curve is given in Figure 11.

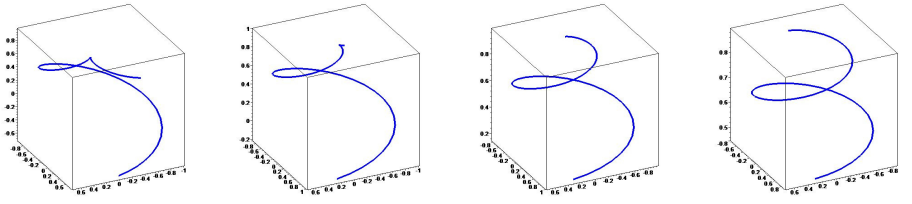


Figure 11: ζ_2-Smarandache curve, $m = \frac{1}{3}, \frac{1}{5}, \frac{1}{8}, \frac{1}{16}$ and $s = [-5, 5]$

Theorem 3.20. The geodesic curvature $K_g^{\zeta_2}$ according to $\zeta_2(s)$-Smarandache curve is

$$K_g^{\zeta_2} = \frac{\sqrt{2}\cot(ns)}{1 - \cot(ns)}.$$

Proof. If we take the derivative of (3.46) and from the equation (2.8) we get

$$T_{\zeta_2}\frac{ds^*}{ds} = \frac{1}{\sqrt{2}}(T_B - \cot(ns)T_B). \tag{3.47}$$

If we take the norm of (3.47), we get

$$\frac{ds^*}{ds} = \frac{\tan(ns) - 1}{\tan(ns)\sqrt{2}}.$$

We obtain the tangent vector of $\zeta_2(s)$-Smarandahce curve as in

$$T_{\zeta_2} = T_B. \tag{3.48}$$

The derivative of (3.48) is

$$T'_{\zeta_2} = \frac{\sqrt{2}(-B + \cot(ns)B \wedge T_B)}{1 - \cot(ns)}. \tag{3.49}$$

and

$$B \wedge T_{\zeta_2} = B \wedge T_B. \tag{3.50}$$

From the equation (2.4), (3.49) and (3.50), the geodesic curvature of ζ_2 is obtained. Q.E.D.

Definition 3.21. Let $\zeta = \zeta(s)$ be a curve and $\{B, T_B, B \wedge T_B\}$ be Sabban frame of this curve. Then $T_B(B \wedge T_B)$-Smarandache curve ($\zeta_3(s)$-Smarandache curve) is given by

$$\zeta_3(s) = \frac{1}{\sqrt{2}}(T_B + (B \wedge T_B)). \tag{3.51}$$

According to equation (2.8) we can parameterize the $\zeta_3(s)$-Smarandache curve as in the following form

$$\begin{aligned}\zeta_3(s) &= \frac{1}{\sqrt{2}}\Big(\cos(s)\cos(ns)+n\sin(s)\sin(ns)+\frac{n}{m}\sin(s),\\ &\quad \sin(s)\cos(ns)-n\cos(s)\sin(ns)-\frac{n}{m}\cos(s),\\ &\quad \frac{n}{m}\sin(ns)-n\Big).\end{aligned}$$

The shape for this curve is given in Figure 12.

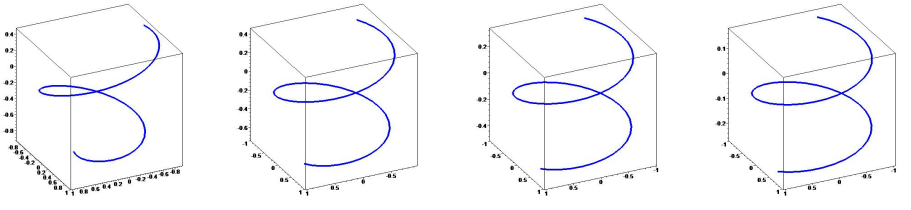


Figure 12: ζ_3-Smarandache curve, $m=\frac{1}{3},\frac{1}{5},\frac{1}{8},\frac{1}{16}$ and $s=[-5,5]$

Theorem 3.22. The geodesic curvature $K_g^{\zeta_3}$ according to $\zeta_3(s)$-Smarandache curve is

$$K_g^{\zeta_3}=\frac{-\cot(ns)(\sigma_{23}+\sigma_{24})}{(1+\cot^2(ns))^{\frac{5}{2}}},$$

where the coefficients σ_{23} and σ_{24} are

$$\begin{aligned}\sigma_{23} &= -1-(\cot(ns))'-3\cot^2(ns)-2\cot^4(ns),\\ \sigma_{24} &= -\cot(ns)+(\cot(ns))'-2\cot^4(ns).\end{aligned}$$

Proof. If we take the derivative of (3.51) and from the equation (2.8) we get

$$T_{\zeta_3}\frac{ds^*}{ds}=\frac{1}{\sqrt{2}}(-B-\cot(ns)T_B+\cot(ns)(B\wedge T_B)). \tag{3.52}$$

If we take the norm of (3.52), we have

$$\frac{ds^*}{ds}=\frac{\sqrt{1+2\cot^2(ns)}}{\sqrt{2}}.$$

We obtain the tangent vector of $\zeta_3(s)$-Smarandahce curve as in

$$T_{\zeta_3}=\frac{-B-\cot(ns)T_B+\cot(ns)B\wedge T_B}{\sqrt{1+\cot^2(ns)}}. \tag{3.53}$$

The derivative of (3.53) is

$$T'_{\zeta_3} = \frac{\sqrt{2}}{(1+2\cot^2(ns))^2}(\sigma_{22}B + \sigma_{23}T_B + \sigma_{24}(B \wedge T_B)). \tag{3.54}$$

and

$$B \wedge T_{\zeta_3} = \frac{-\cot(ns)T_B - \cot(ns)B \wedge T_B}{\sqrt{1+2\cot^2(ns)}} \tag{3.55}$$

where the coefficients σ_{22}, σ_{23}, σ_{24} are

$$\begin{aligned}
\sigma_{22} &= 2\cot(ns)(\cot(ns))' + \cot(ns) + 2\cot^3(ns),\\
\sigma_{23} &= -1 - (\cot(ns))' - 3\cot^2(ns) - 2\cot^4(ns),\\
\sigma_{24} &= -\cot(ns) + (\cot(ns))' - 2\cot^4(ns).
\end{aligned}$$

From the equation (2.4), (3.54) and (3.55), the geodesic curvature of ζ_3 is obtained. Q.E.D.

Definition 3.23. Let $\zeta = \zeta(s)$ be a curve and $\{B, T_B, B \wedge T_B\}$ be Sabban frame of this curve. Then $BT_B(B \wedge T_B)$-Smarandache curve ($\zeta_4(s)$-Smarandache curve) is given by

$$\zeta_4(s) = \frac{1}{\sqrt{3}}(B + T_B + (B \wedge T_B)). \tag{3.56}$$

According to equation (2.8) we can parameterize the $\zeta_4(s)$-Smarandache curve as in the following form

$$\begin{aligned}
\zeta_4(s) = \frac{1}{\sqrt{3}}\Big(&\cos(s)(\cos(ns) - \sin(ns)) + n\sin(s)(\cos(ns) + \sin(ns))\\
&+\frac{n}{m}\sin(s), \sin(s)(\cos(ns) - \sin(ns)) - n\cos(s)(\cos(ns) + \sin(ns))\\
&-\frac{n}{m}\cos(s), \frac{n}{m}(\cos(ns) + \sin(ns)) - n\Big).
\end{aligned}$$

The shape for this curve is given in Figure 13.

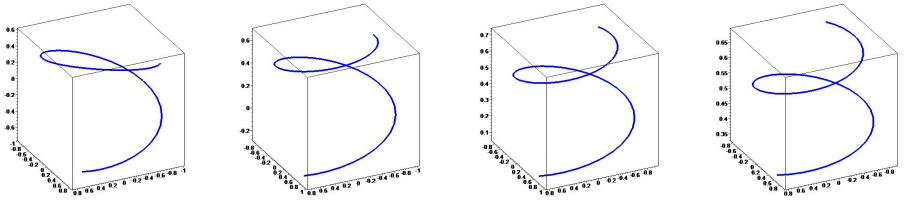


Figure 13: ζ_4-Smarandache curve, $m = \frac{1}{3}, \frac{1}{5}, \frac{1}{8}, \frac{1}{16}$ and $s = [-5, 5]$

Theorem 3.24. The geodesic curvature $K_g^{\zeta_4}$ according to $\zeta_4(s)$-Smarandache curve is

$$K_g^{\zeta_4} = \frac{-\cot(ns)\sigma_{26} + (1-\cot(ns)\sigma_{27}}{(2-\cot(ns)+2\cot^2(ns))^{\frac{5}{2}}}.$$

where the coefficients σ_{26} and σ_{27} are

$$\begin{aligned}
\sigma_{26} &= -(\cot(ns))' - \cot(ns)(\cot(ns))' - 2 - 4\cot^2(ns) + 2\cot(ns) + 2\cot^3(ns) - 2\cot^4(ns), \\
\sigma_{27} &= \cot(ns)(\cot(ns))' + 2\cot(ns) - 4\cot^2(ns) + 2(\cot(ns))' + 4\cot^3(ns) - 2\cot^4(ns).
\end{aligned}$$

Proof. If we take the derivative of (3.56) and from the equation (2.8) we get

$$T_{\zeta_4}\frac{ds^*}{ds} = \frac{1}{\sqrt{3}}(-B + (1-\cot(ns))T_B + \cot(ns)B \wedge T_B) \tag{3.57}$$

If we take the norm of (3.57), we get

$$\frac{ds^*}{ds} = \frac{\sqrt{2-2\cot(ns)+2\cot^2(ns)}}{\sqrt{3}}.$$

We obtain the tangent vector of $\zeta_4(s)$-Smarandahce curve as in

$$T_{\zeta_4} = \frac{-B + (1-\cot(ns))T_B + \cot(ns)B \wedge T_B}{\sqrt{2-2\cot(ns)+2\cot^2(ns)}}. \tag{3.58}$$

The derivative of (3.58) is

$$T'_{\zeta_4} = \frac{\sqrt{3}(\sigma_{25}B + \sigma_{26}T_B + \sigma_{27}B \wedge T_B)}{(2-2\cot(ns)+2\cot^2(ns))^2} \tag{3.59}$$

and

$$B \wedge T_{\zeta_4} = \frac{-\cot(ns)T_B + (1-\cot(ns))B \wedge T_B}{\sqrt{2-2\cot(ns)+2\cot^2(ns)}} \tag{3.60}$$

where the coefficients σ_{25}, σ_{26}, σ_{27} are

$$\begin{aligned}
\sigma_{25} &= -(\cot(ns))' + 2\cot(ns)(\cot(ns))' - 2 + 4\cot(ns) - 4\cot^2(ns) + 2\cot^3(ns) \\
\sigma_{26} &= -(\cot(ns))' - \cot(ns)(\cot(ns))' - 2 - 4\cot^2(ns) + 2\cot(ns) + 2\cot^3(ns) - 2\cot^4(ns), \\
\sigma_{27} &= \cot(ns)(\cot(ns))' + 2\cot(ns) - 4\cot^2(ns) + 2(\cot(ns))' + 4\cot^3(ns) - 2\cot^4(ns).
\end{aligned}$$

From the equation (2.4), (3.59) and (3.60), the geodesic curvature of ζ_4 is obtained. Q.E.D.

4 Acknowledgement

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